Calculus I Exam #2
Sections 3.6 – 4.4
October 22, 2009

Name Key

Make sure you show all your work on this paper. Solutions without correct supporting work will not be accepted. **No calculators may be used on this exam.** Good luck!

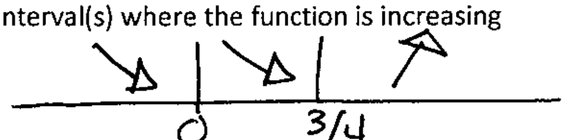
1. Given $f(x) = x^4 - x^3$, find: (10 points)

   a. Critical Numbers (You do not have to find the y-coordinate of the critical points but make sure you indicate whether you will have a relative maximum, minimum, or neither at each critical number.)

   $f'(x) = 4x^3 - 3x^2$
   $= x^2(4x-3)$

   $\boxed{\text{CN: } \underbrace{x=0}_{\text{Neither}},\ \underbrace{x=3/4}_{\text{Rel. min}}}$

   b. Interval(s) where the function is increasing

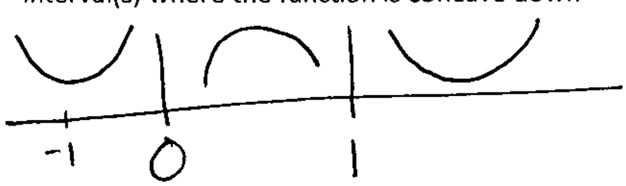


   $\boxed{\text{Inc: } (3/4, \infty)}$

   c. Inflection Points (You do not have to find the y-coordinate of the points.)

   $f''(x) = 12x^2 - 6x = 12x(x - 1/2)$ $\boxed{x=0,\ x=1/2}$

   d. Interval(s) where the function is concave down



   $\boxed{\text{Con Down: } (0, 1/2)}$

2. Locate the **absolute** extrema of $f(x) = x^3 - \frac{3}{2}x^2$ on the **closed** interval $[-1,2]$. (8 points)

   $f'(x) = 3x^2 - 3x = 3x(x-1)$
   $x = 0,\ x = 1$

   $f(0) = 0$
   $f(1) = 1 - \frac{3}{2} = -1/2$
   $f(-1) = -1 - \frac{3}{2} = -5/2$
   $f(2) = 8 - 6 = 2$

   $\boxed{\begin{array}{l} \text{Abs. Max} = 2 \text{ @ } x = 2 \\ \text{Abs. Min} = -5/2 \text{ @ } x = -1 \end{array}}$

3. Determine whether the Mean Value Theorem can be applied to $f(x) = x^{2/3}$ on the interval $[0,1]$. If it can be applied find all values of c guaranteed by the theorem. (8 points)

   f is Cont on $[0,1]$ ✓ f is diff on $(0,1)$ ✓

   $f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3\sqrt[3]{x}}$

   $\frac{2}{3\sqrt[3]{x}} = \frac{1-0}{1}$

   $2 = 3\sqrt[3]{x}$

   $\frac{2}{3} = \sqrt[3]{x}$

   $x = 8/27$

   $\boxed{c = 8/27}$

4. Find the critical numbers of $f(x) = x^{2/3} + x$ and determine if the graph of the function has a horizontal tangent line or a vertical tangent line at each critical number. (8 points)

$$f'(x) = \frac{2}{3}x^{-1/3} + 1 = \frac{2}{3\sqrt[3]{x}} + 1 = \frac{2 + 3\sqrt[3]{x}}{3\sqrt[3]{x}}$$

CN: $f'(x) = 0$

$2 + 3\sqrt[3]{x} = 0$

$\sqrt[3]{x} = -2/3$

$\boxed{x = -8/27 \rightarrow \text{Horiz. Tan Line}}$

$f'(x)$ DNE

$\boxed{x = 0 \rightarrow \text{Vertical Tan Line}}$

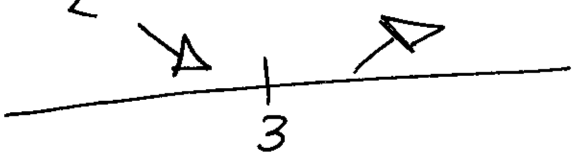
5. Find the horizontal asymptote(s) of $f(x) = \frac{3x^4+7x-5}{2x^4+1}$, if they exist. (6 points)

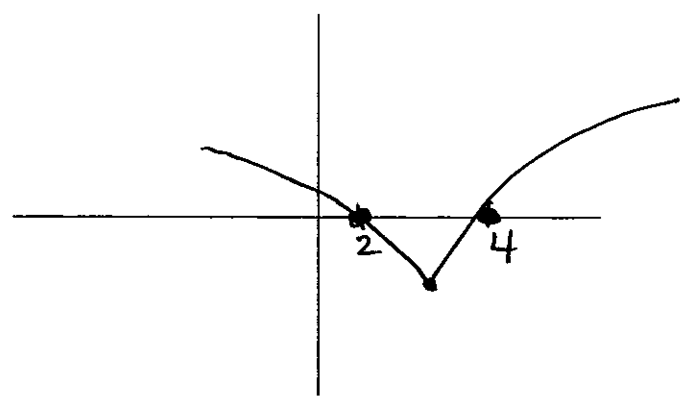
$$\lim_{x \to \infty} \frac{3 + 7/x^3 - 5/x^4}{2 + 1/x^4} = 3/2$$

$\boxed{\text{HA: } y = 3/2}$

6. Sketch the graph of a continuous function that satisfies the criteria given: (10 points)

$f(2) = f(4) = 0$
$f'(x) > 0$ if $x > 3$ and $f'(x) < 0$ if $x < 3$
$f'(3)$ does not exist
$f''(x) < 0$ if $x \neq 3$





7. Find the inflection points of $f(x) = \sin x - \cos x$ on the interval $[0, 2\pi]$. (8 points)

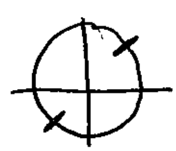
$f'(x) = \cos x + \sin x$

$\cos x = -\sin x$

$x = 3\pi/4,\ 7\pi/4$

CN

$f''(x) = -\sin x + \cos x$



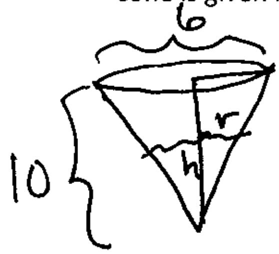
$\cos x = \sin x$

$x = \pi/4,\ 5\pi/4$

$0 \quad \pi/4 \quad 5\pi/4 \quad \pi$

$\boxed{\text{IP: } (\pi/4, 0),\ (5\pi/4, 0)}$

8. A student is using a straw to drink from a conical paper cup, whose axis is vertical, at a rate of 3 cubic centimeters per second. If the height of the cup is 10 centimeters, and the diameter of its opening is 6 centimeters, how fast is the level of the liquid falling when the depth of the liquid is 5 centimeters? Recall that the volume of a right-circular cone is given by $V = \frac{1}{3}\pi r^2 h$. (10 points)



$$\frac{dV}{dt} = -3 \text{ cc/sec}$$

$$\frac{dh}{dt} = ? \text{ when } h = 5\text{cm}$$

$$\frac{r}{h} = \frac{3}{10}$$

$$r = \frac{3}{10}h$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{3h}{10}\right)^2 h$$

$$V = \frac{3\pi}{100}h^3$$

$$\frac{dV}{dt} = \frac{9\pi}{100}h^2\frac{dh}{dt}$$

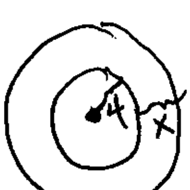
$$\frac{dV}{dt} = \frac{9\pi}{100}(5)^2\frac{dh}{dt}$$

$$-3 = \frac{9\pi}{4}\frac{dh}{dt}$$

$$\frac{dh}{dt} = -3 \cdot \frac{4}{9\pi}$$

$$\boxed{\frac{dh}{dt} = -\frac{4}{3\pi} \text{ cm/sec}}$$

9. A spherical iron ball 8 in. in diameter is coated with a layer of ice of uniform thickness. If the ice melts at the rate of 10 in^3/min, how fast is the thickness of the ice decreasing when it is 2 in. thick? (Recall that the surface area of sphere is given by $S = 4\pi r^2$ and the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.) (10 points)



$$\frac{dV}{dt} = -10 \text{ in}^3/\text{min}$$

$$\frac{dx}{dt} = ? \text{ when } x = 2 \text{ in}$$

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3}\pi(x+4)^3$$

$$\frac{dV}{dt} = 4\pi(x+4)^2\frac{dx}{dt}$$

$$-10 = 4\pi(6)^2\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{-10}{36(4)\pi} = \boxed{\frac{-5}{72\pi} \text{ in/min}}$$

10. Find the derivative: $y = \tan(\arcsin t)$
(6 points)

$$\boxed{y' = \frac{\sec^2(\arcsin t)}{\sqrt{1-t^2}}}$$

11. Write the equation of the tangent line when $x = 2$ for the function $y = (x-1)^x$. (10 points)

$x = 2 \to y = 1^2 = 1$
$(2,1)$ 1pt

$$\ln y = x \ln(x-1)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{x}{x-1} + \ln(x-1)$$

$$\boxed{\frac{dy}{dx} = (x-1)^x\left[\frac{x}{x-1} + \ln(x-1)\right]}$$ 7pts

$$m = \left.\frac{dy}{dx}\right|_{x=2} = 1^2\left[\frac{2}{1} + \ln 1\right] = 2$$ 1pt

$$y - 1 = 2(x-2)$$

$$\boxed{y = 2x - 3}$$ 1pt

12. Find dy: $y = \cosh(2\ln x)$ (6 points)

$$\boxed{\frac{dy}{dx} = \sinh(2\ln x) \cdot \frac{2}{x}}$$