Calculus I Quiz #5
Thursday, November 5, 2009

Name Key

Show all your work on this paper. Solutions without correct supporting work will not be accepted.

1. Evaluate $\int \left(4x^6 + \sqrt[3]{x^2} - \frac{1}{x} + \sin x\right) dx$ (4 points)

$$\boxed{\frac{4}{7}x^7 + \frac{3}{5}x^{5/3} - \ln|x| - \cos x + C}$$

2. Evaluate: (4 points each)

a. $\lim_{x\to 0} \frac{\sin \pi x}{\sin 2\pi x}$

$$= \lim_{x\to 0} \frac{\pi \cos \pi x}{2\pi \cos 2\pi x}$$

$$= \boxed{1/2}$$

b. $\lim_{x\to\infty} (1 + \frac{1}{x})^x$ $\longrightarrow$ $= \boxed{e}$

$$y = \left(1 + \frac{1}{x}\right)^x$$

$$\ln y = x \ln\left(1 + \frac{1}{x}\right)$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(1 + 1/x)}{1/x}$$

$$= \lim_{x\to\infty} \frac{\frac{x}{x+1} \cdot -1/x^2}{-1/x^2} = \lim_{x\to\infty} \frac{x}{x+1}$$

$$= 1.$$

LOOK ON THE BACK

4. A silo (base not included) is to be constructed in the form of a cylinder surmounted by a hemisphere. The cost of construction is \$2/square unit for the cylindrical sidewall and \$4/square unit for the hemisphere. Determine the dimensions to be used if the volume must be 1000 cubic units and the cost of construction is to be kept to a minimum. Neglect the thickness of the silo and the waste in construction. (8 points)

$$V = 1000 \text{ units}^3 = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$$

$$1000 = \pi r^2 h + \frac{2}{3}\pi r^3$$

$$\pi r^2 h = 1000 - \frac{2}{3}\pi r^3$$

Minimize Cost

$$h = \frac{1000}{\pi r^2} - \frac{2\pi r^3}{3\pi r^2} = \frac{1000}{\pi} r^{-2} - \frac{2}{3} r$$

$$C = \$2(2\pi r h) + \$4(2\pi r^2)$$

$$C = 4\pi r\left(\frac{1000}{\pi} r^{-2} - \frac{2}{3} r\right) + 8\pi r^2$$

$$C = 4000 r^{-1} - \frac{8\pi}{3} r^2 + 8\pi r^2$$

$$\boxed{C = 4000 r^{-1} + \frac{16\pi}{3} r^2}$$

$$C' = -4000 r^{-2} + \frac{32\pi}{3} r \rightarrow C'' = 8000 r^{-3} + \frac{32\pi}{3}$$

$$C''\left(\sqrt[3]{\frac{375}{\pi}}\right) = \frac{8000}{\left(\sqrt[3]{\frac{375}{\pi}}\right)^3} + \frac{32\pi}{3}$$

$$> 0 \quad \therefore \text{Min}$$

$$C' = \frac{-12000 + 32\pi r^3}{3r^2}$$

CN: $\cancel{r = 0}$

$$32\pi r^3 = 12000$$

$$r^3 = \frac{12000}{32\pi}$$

$$r^3 = \frac{375}{\pi}$$

$$\boxed{r = \sqrt[3]{\frac{375}{\pi}}} = 5\sqrt[3]{\frac{3}{\pi}}$$

Dimensions to minimize Cost: $r = \sqrt[3]{\frac{375}{\pi}}$; $h = \frac{750}{\pi\left(\sqrt[3]{\frac{375}{\pi}}\right)^2}$

$$h = \frac{1000}{\pi\left(\sqrt[3]{\frac{375}{\pi}}\right)^2} - \frac{2}{3}\sqrt[3]{\frac{375}{\pi}}$$

$$= \frac{1000 \cdot 3 - 2\sqrt[3]{\frac{375}{\pi}}\left(\sqrt[3]{\frac{375}{\pi}}\right)^2 \cdot \pi}{3\pi\left(\sqrt[3]{\frac{375}{\pi}}\right)^2} = \frac{3000 - 2\pi\left(\frac{375}{\pi}\right)}{3\pi\left(\sqrt[3]{\frac{375}{\pi}}\right)^2} = \boxed{\frac{750}{\pi\left(\sqrt[3]{\frac{375}{\pi}}\right)^2}} = h$$

OK if Simplified More