

Implicit Differentiation

Section 3.6

Implicit vs. Explicit

- ✓ y is written **explicitly** as a function of x when y is isolated on one side of the equation.
- ✓ When y is not isolated we say the function is written **implicitly**.

Why do we need implicit differentiation?

- ✓ All of the functions you have seen so far in this class could easily be written explicitly. Whenever this is the case, you don't need to use implicit differentiation. But what do you do if it is very difficult or impossible to isolate y ? This is when we need implicit differentiation.

✓ Example: Solve $x^2y + y^2x = -2$ for y .

Keep In Mind



- ✓ When finding dy/dx and you are differentiating a term involving x alone, you can differentiate as usual.
- ✓ When you differentiate terms involving y , you must apply the Chain Rule, because you are assuming that y is defined implicitly as a function of x .

Guidelines for Implicit Differentiation

- ✓ Differentiate both sides of the equation *with respect to x* .
- ✓ Collect all terms involving dy/dx on the left side of the equation and move all other terms to the right side of the equation.
- ✓ Factor dy/dx out of the left side of the equation.
- ✓ Isolate dy/dx by dividing.

Examples

- ✓ $x^3 + y^4 = 7y - 5$
- ✓ $x^2y + y^2x = -2$
- ✓ $2 \sin x \cos y = 1$
- ✓ Find the normal line to the curve $y = 2\sin(\pi x - y)$ at the point $(1, 0)$

Finding a 2nd Derivative Implicitly

- ✓ Find dy/dx as before.
- ✓ Differentiate both sides to get the 2nd derivative. You will have to use implicit differentiation on the right-hand side.
- ✓ Substitute your answer to the first step for dy/dx .
- ✓ If possible, substitute in the original equation to simplify your answer.

Examples

Find d^2y/dx^2 .

✓ $xy + y^2 = 1$