Derivatives of Inverse Functions

Sections 3.7 and 3.8

Continuity & Differentiability of Inverse Functions

Let *f* be a function whose domain is an interval *I*. If *f* has an inverse, then the following statements are true.

1. If *f* is continuous on its domain, then f^{-1} is continuous on its domain.
2. If *f* is differentiable at *c* and $f'(c) \neq 0$, then f^{-1} is differentiable at $f(c)$.

The Derivative of an Inverse Function

Let *f* be a function that is differentiable on an interval *I*. If *f* has an inverse function *g*, then *g* is differentiable at any *x* for which $f'(g(x)) \neq 0$. Moreover, $g'(x) = 1/f'(g(x))$.

Other Basic Derivatives

Natural Logarithmic Function

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

Bases Other than *e*

$$\frac{d}{dx}[a^x] = (\ln a)a^x$$

$$\frac{d}{dx}[\log_a x] = \frac{1}{(\ln a)x}$$

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Assignment for next time

Section 3.6 Homework

Section 3.8, ALL of #1-20 WITHOUT using your calculator AT ALL. Do not even get it out of your bag.

Section 3.7 through #40 on the assignment sheet

Logarithmic Differentiation

Logarithmic differentiation is helpful when we have a function that could easily be rewritten using the basic properties of logarithms.

Messy functions involving radicals and quotients are good candidates.

We really logarithmic differentiation to differentiate functions where we have a base and exponent which both involve our variable.

Steps for Logarithmic Differentiation

- Take ln of both sides.
- Use the properties of logarithms to rewrite the right-hand side.
- Differentiate using implicit differentiation.
- Simplify and isolate *dy/dx*.
- Use the original equation to substitute for *y*.
- Simplify.

Examples

$y = x^{x-1}$

$y = (1+x)^{1/x}$

$y = \sqrt[3]{\dfrac{x^2+1}{x^2-1}}$

Derivatives of Inverse Trigonometric Functions

$$\frac{d}{dx}[\arcsin x] = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}[\arccos x] = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}[\arctan x] = \frac{1}{1+x^2} \qquad \frac{d}{dx}[arc\cot x] = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}[arc\sec x] = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}[arc\csc x] = \frac{-1}{|x|\sqrt{x^2-1}}$$