Calculus I Exam #1
Fall 2009

Name Key

You must show all of your work on this paper. ***Solutions without correct supporting work will not be accepted.*** All answers should be exact unless stated otherwise in the problem. **All answers must be exact.**

1. Find the limit *analytically* or explain why it does not exist. Write ∞ or -∞ when appropriate. (5 pts each)

a. $\lim_{x\to 1}\frac{1-\sqrt{x}}{1-x}$ = $\frac{1}{2}$

$$\lim_{x\to 1}\left(\frac{1-\sqrt{x}}{1-x}\right)\left(\frac{1+\sqrt{x}}{1+\sqrt{x}}\right)=\lim_{x\to 1}\frac{1}{1+\sqrt{x}}$$

b. $\lim_{x\to -\infty}\frac{x^2-4x+8}{3x^3}$ = 0

$$=\lim_{x\to -\infty}\frac{\frac{1}{x}-\frac{4}{x^2}+\frac{8}{x^3}}{3}$$

$$=0$$

c. $\lim_{x\to 5}\cos\left(\frac{\pi x}{6}\right)$ = $-\frac{\sqrt{3}}{2}$

$$=\cos\frac{5\pi}{6}$$

d. $\lim_{x\to\infty} e^{1/x}\cos\frac{1}{x}$ = 1

$$=e^0\cos 0$$

2. State the 3 conditions that must hold for a function $y = f(x)$ to be continuous at $x = 2$. Be very precise and use correct notation. (6 points)

1) $f(2)$ must exist

2) $\lim_{x\to 2} f(x)$ must exist

3) $\lim_{x\to 2} f(x) = f(2)$

3. Given the function $f(x) = \frac{x+2}{x^2-x-6}$, find all points (if any) where the function is discontinuous and classify the discontinuities as removable or nonremovable. (6 points)

$$f(x)=\frac{x+2}{(x-3)(x+2)}$$

$x=-2 \rightarrow$ Removable
$x=3 \rightarrow$ Nonremovable

4. Given $\lim_{x\to -2}(2x-3) = -7$, find an open interval about $x_0 = -2$ on which the inequality $|f(x) - L| < \epsilon$ holds for $\epsilon = 0.02$. Then give a value for $\delta > 0$ such that for all x satisfying $0 < |x - x_0| < \delta$ the inequality $|f(x) - L| < \epsilon$ holds. (6 points)

$$|2x-3+7| < .02$$
$$|2x+4| < .02$$
$$|x+2| < .01$$

Interval about $x_0 = -2$:

$$(-2.01, -1.99)$$

$$\delta = .01$$

5. Find the derivative but ***do not simplify*** your answers. (6 points each)

a. $f(x) = x^5 + \sqrt[5]{x^2} - \frac{1}{x^3} + \pi^8$

$$\boxed{f'(x) = 5x^4 + \frac{2}{5}x^{-3/5} + 3x^{-4}}$$

b. $y = \sin(\theta + \sqrt{3\theta + 1})$

$$\boxed{y' = \cos(\theta + \sqrt{3\theta+1})\left[1 + \tfrac{1}{2}(3\theta+1)^{-1/2}(3)\right]}$$

c. $g(t) = \frac{\sec t}{t^2 - 5}$

$$\boxed{g'(t) = \frac{\overbrace{(t^2-5)\sec t\tan t}^{2pt} - \overbrace{(\sec t)(2t)}^{2pt}}{\underbrace{(t^2-5)^2}_{2pt}}}$$

-1 for missed sign

-6 if quotient rule not attempted

6. Find an equation for the tangent line to the graph of $f(x) = x^2 e^{3x}$ at the point where $x = 1$. You may leave your answer in point-slope form. (8 points)

$$f(1) = e^3$$

$$f'(x) = x^2 \cdot 3e^{3x} + 2xe^{3x}$$
$$f'(1) = 3e^3 + 2e^3 = 5e^3$$

$$\boxed{y - e^3 = 5e^3(x-1)}$$

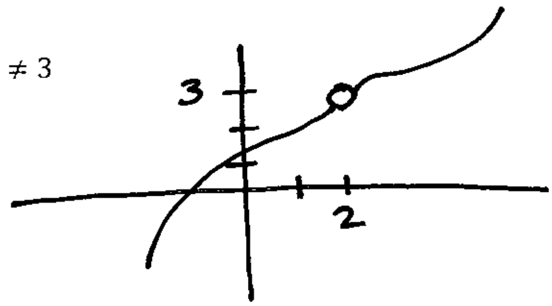
7. Find an equation for the line tangent to the curve given by $x = t,\ \ y = \sqrt{t}$ at the point where $t = 1/9$. You may leave your answer in point-slope form. (8 points)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{1}{2}t^{-1/2}}{1} = \frac{1}{2\sqrt{t}}\Big|_{t=1/9} = \frac{1}{2(\frac{1}{3})} = \frac{3}{2}$$

$$\left(\tfrac{1}{9}, \tfrac{1}{3}\right) \to \boxed{y - \tfrac{1}{3} = \tfrac{3}{2}\left(x - \tfrac{1}{9}\right)}$$

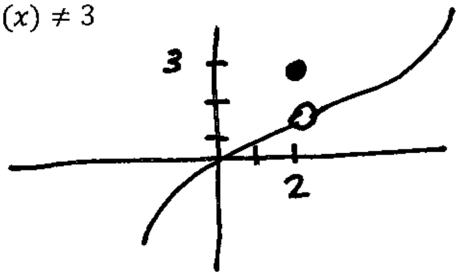
8. Sketch the graph of a function that satisfies the criteria below. If it is not possible, explain why. (4 pts each)

a. $\lim_{x\to 2} f(x) = 3$ and $f(2) \neq 3$



Answers are not unique - yours can be correct & not look like mine!

b. $f(2) = 3$ and $\lim_{x\to 2} f(x) \neq 3$



9. The number of gallons of water in a tank t minutes after the tank has started to drain is $Q(t) = 200(30-t)^2$.
(5 points each)

a. How fast is the water running out at the end of 10 min?

$Q'(t) = -400(30-t)$ $\qquad$ $Q'(10) = -400(30-10) = \boxed{-8000 \text{ gal/min}}$

$\boxed{\text{Speed} = 8000 \text{ gal/min}}$

b. What is the average rate at which the water flows out during the first 10 min?

$$\frac{Q(10)-Q(0)}{10-0} = \frac{200(20)^2 - 200(30)^2}{10}$$

$$= -10{,}000 \text{ gal/min}$$

$\boxed{\text{Speed} = 10{,}000 \text{ gal/min}}$

10. ***Use the definition*** of the derivative to find $\frac{dy}{dx}$ for $y = \frac{2}{3x+5}$. (10 points)

$$y' = \lim_{h\to 0} \frac{\frac{2}{3(x+h)+5} - \frac{2}{3x+5}}{h}$$

$$= \lim_{h\to 0} \frac{2(3x+5) - 2(3x+3h+5)}{h(3x+5)(3x+3h+5)}$$

$$= \lim_{h\to 0} \frac{6x+10-6x-6h-10}{h(3x+5)(3x+3h+5)}$$

$$= \lim_{h\to 0} \frac{-6}{(3x+5)(3x+3h+5)} = \boxed{\frac{-6}{(3x+5)^2}}$$