Calculus I Quiz #3
Sections 3.6 – 3.9
Fall 2009

Name Key

All answers must be exact. Make sure you show all your work on this paper.

1. Find $\frac{dy}{dx}$: $y = \arcsin(2x^5)$ (3 points)

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x^5)^2}} \cdot 10x^4 = \boxed{\frac{10x^4}{\sqrt{1-4x^{10}}}}$$

2. $xy + y^2 = 1$ (3 points each)

a. Find $\frac{dy}{dx}$.

$$x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -y$$

$$\boxed{\frac{dy}{dx} = \frac{-y}{x+2y}}$$

b. Find the normal line to the curve at the point (0,-1).

$$\left.\frac{dy}{dx}\right|_{(0,-1)} = \frac{1}{0-2} = -1/2$$

$$m_{norm} = 2$$

$$\boxed{y = 2x - 1}$$

c. Find $\frac{d^2y}{dx^2}$.

$$\frac{d^2y}{dx^2} = \frac{(x+2y)(-1)\frac{dy}{dx} + y\left(1+2\frac{dy}{dx}\right)}{(x+2y)^2}$$

$$= \frac{-(x+2y)\left(\frac{-y}{x+2y}\right) + y + 2y\left(\frac{-y}{x+2y}\right)}{(x+2y)^2}$$

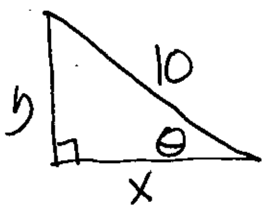
$$= \frac{y(x+2y) + y(x+2y) - 2y^2}{(x+2y)^3} = \boxed{\frac{2xy+2y^2}{(x+2y)^3}}$$

$$= \frac{xy+2y^2+xy+2y^2-2y^2}{(x+2y)^3}$$

LOOK ON THE BACK

3. A 10-ft ladder is leaning against a building. If the bottom of the ladder is sliding along the pavement directly away from the building at 2 ft/sec, (4 points each)

(a) how fast is the top of the ladder moving down when the foot of the ladder is 3 feet from the wall?



$\frac{dx}{dt} = 2 \text{ ft/sec} \qquad \frac{dy}{dt} = ? \text{ when } x = 3 \text{ ft}$

$x^2 + y^2 = 10^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

$y = \sqrt{91}$

$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt} = -\frac{3}{\sqrt{91}}(2) = \boxed{\frac{-6}{\sqrt{91}} \text{ ft/sec}}$

(b) At what rate is the angle between the ladder and the ground changing when the foot of the ladder is 3 feet from the wall?

$\frac{d\theta}{dt} = ?$

$\cos\theta = \frac{x}{10}$

$10\cos\theta = x$

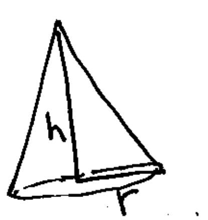
$-10\sin\theta \frac{d\theta}{dt} = \frac{dx}{dt}$

$-10\left(\frac{\sqrt{91}}{10}\right)\frac{d\theta}{dt} = 2$

$\boxed{\frac{d\theta}{dt} = \frac{-2}{\sqrt{91}} \text{ rad/s}}$

4. Sand falls from conveyor belt at a rate of 14 m^3/min onto the top of a conical pile. The height of the pile is always 3/8 of the diameter of the base. How fast is the height changing when the pile is 7 m high? (4 points) (Recall that the volume of a cone is $\frac{1}{3}\pi r^2 h$)

$\frac{dV}{dt} = 14 \text{ m}^3/\text{min}, \quad \frac{dh}{dt} = ? \text{ when } h = 7\text{m}$



$h = \frac{3}{8}(2r)$

$h = \frac{3}{4}r$

$r = \frac{4}{3}h$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h$

$V = \frac{16\pi}{27}h^3$

$\frac{dV}{dt} = \frac{16\pi}{9}h^2\frac{dh}{dt} = \frac{16\pi}{9}(7)^2\frac{dh}{dt}$

$14 = \frac{16\pi(49)}{9}\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{14(9)}{16\pi(49)}$

$\boxed{\frac{dh}{dt} = \frac{9}{56\pi} \text{ m/min}}$