

An infinite series is a sum of the form $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + \dots + a_n + \dots$

The n th partial sum S_n is the sum of the first n terms of the series. $S_n = a_1 + a_2 + \dots + a_n$

If the sequence of partial sums converges to S , then the series $\sum_{n=1}^{\infty} a_n$ is said to converge to or equal S . If the series of partial sums diverges, then so does the series.

One way to determine whether a series converges or diverges is to find a formula for the n th partial sum and investigate its limit.

Example

As you can see this method is tedious under the best of circumstances. We will spend most of this chapter developing methods by which we can determine whether a given series converges or diverges without having to resort to the partial sum definition.

To begin, we define what is known as a GEOMETRIC SERIES

$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + \dots + ar^n + \dots, a \neq 0$ is a geometric series with ratio r .

Examples:

Geometric Series Test

A geometric series with ratio r diverges if $|r| \geq 1$. If $0 < |r| < 1$, then the series converges to the sum $S = \frac{a}{1-r}$.

Keep in mind that a is simply the first term of the series and r is the ratio.

Examples:

Nth Term Test for Divergence

8.2.2

If $\{a_n\}$ does not converge to 0, then the series $\sum a_n$ diverges.

Note that this is a test for divergence only. You can never say that a series converges by the nth term test. Furthermore, it is NOT an if and only if statement. Just because $\{a_n\}$ does converge to 0 does not mean that the corresponding series converges. You would need to do more investigation.

Examples:

Properties of Infinite Series

If $\sum a_n = A$ and $\sum b_n = B$ and c is a real number, then the following series converge to the indicated sums.

1. $\sum ca_n = cA$

2. $\sum (a_n + b_n) = A + B$

3. $\sum (a_n - b_n) = A - B$

Examples: