8.5, Alternating Series

exercises 9, 10, 11, 12, 13, 15, 20, 22, 23, 24, 37, 38, 39, 40, 44, 46, 51,

Up to this point most of our series in question have been positive-term series. In the next few sections we will consider the convergence and divergence of series with both positive and negative terms. The first example is:

THE ALTERNATING SERIES

An alternating series is one whose terms alternate in signs.

Example:

To determine the convergence or divergence of an alternating series we use

The Alternating Series Test

Let $a_n > 0$. The alternating series $\sum_{n=1}^{\infty}(-1)^n a_n$ and $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ converge if the following conditions are met:

1) $\lim_{n\to\infty} a_n = 0$ Notice we do not look at the alternating signs for this part.

2) $a_{n+1} \le a_n$ for all n. This just means that the sequence a_n is a decreasing sequence. (It doesn't really have to be true for all n starting at n=1 as long as the sequence is eventually decreasing.

Example:

$$\sum_{n=1}^{\infty}(-1)^n \frac{1}{n}$$

$$\sum_{n=1}^{\infty}\frac{(-1)^n n^3}{2n^3-5}$$

For some series with positive and negative terms, its absolute value actually converges. If this is true, then the series with positive and negative terms also converges and we say the series is **Absolutely Convergent**

Definition:

If the series $\sum |a_n|$ converges, then the series $\sum a_n$ also converges and the series is called **Absolutely Convergent**. If the original series $\sum a_n$ converges, but its absolute value, $\sum |a_n|$, diverges, the series is called **conditionally convergent**

Example:

Determine whether the following series converge conditionally, absolutely, or diverge.

#38

#46

#49