Calculus II Exam #1 (7.1-8.8)
Fall 2008 Name _____

Show all your work on notebook paper. Solutions without correct supporting work will not be accepted. Please BE NEAT and circle your answers. Illegible work will not be graded. There is a 5 point bonus built in. Good Luck!

1. Given the functions $f(x) = -x^2 + 2x$ and $g(x) = x$, set up (but do not evaluate) the integral(s) to find: (8 points each)
   a. the area bounded by the curves

   b. the volume when the bounded region is revolved around the x-axis

   c. the volume when the bounded region is revolved around the line $x = 5$

2. Set up, but do not evaluate, the integral(s) to find the work done in compressing a 15-inch spring from a length of 10 inches to a length of 6 inches if a force of 5 pounds compresses the spring a total of 4 inches. (9 points)

3. Consider a spherical water tank whose radius is 10 ft and whose center is 50 ft above the ground. Set up, but do not evaluate the integral to find the work required to fill this tank by pumping water up from ground level. (9 points)

4. A tank on the wing of a jet aircraft is formed by revolving the region bounded by the graph of $y = \frac{1}{8}x^2\sqrt{2-x}$ and the x-axis about the x-axis where x and y are measured in meters. Find the tank's volume. (9 points)

5. $\int \frac{8x^3 + 13x}{(x^2+2)^2}\,dx$ (9 points)

6. Evaluate: $\lim_{x\to 0^+} (\sin x)^x$ (9 points)

7. Evaluate: $\int \theta \sec\theta \tan\theta \, d\theta$ (9 points)

8. Evaluate: $\int \frac{1}{x\sqrt{4x^2+9}}\,dx$ (9 points)

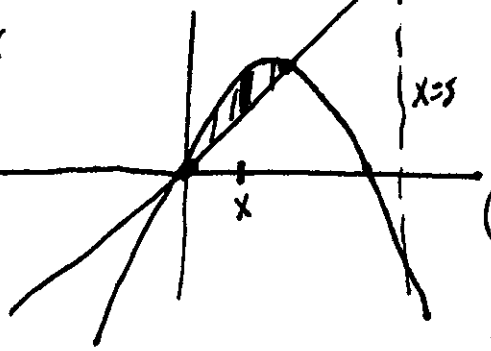
9. $\int \sin^2(3x)\,dx$ (9 points)

10. $\int \sec^3 x \, dx$ (9 points)

Calculus II Exam #1 Key
Fall 2008

1. $f(x) = -x^2+2x$, $g(x) = x$

$-x^2+2x = x$
$x^2 - x = 0$
$x(x-1) = 0$



(a) $A = \int_0^1 (-x^2+2x-x)\,dx$

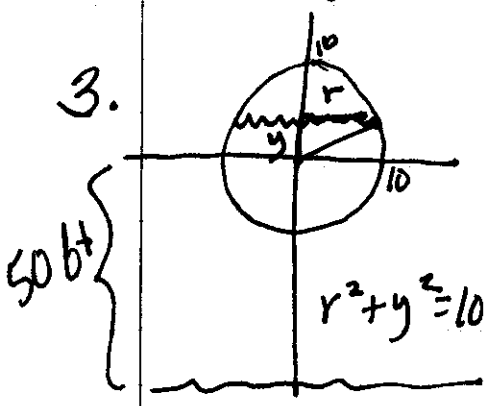
(b) $V = \pi \int_0^1 \left[(-x^2+2x)^2 - x^2\right] dx$

(c) $V = 2\pi \int_0^1 (-x^2+2x-x)(5-x)\,dx$

2. $F = kd$
$5 = k(4)$
$k = \frac{5}{4}$

$W = \int_5^9 \frac{5}{4} x \, dx$

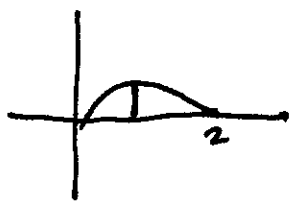
3.



$r^2 + y^2 = 10^2$

$A = \pi r^2$
$A = \pi(100 - y^2)$

$W = \int_{-10}^{10} 62.4\,(50+y)\,\pi\,(100-y^2)\,dy$

4. 

$$V = \pi\int_0^2 \left(\frac{1}{8}x^2\sqrt{2-x}\right)^2 dx$$
$$= \pi\int_0^2 \frac{1}{64}x^4(2-x)\,dx$$
$$= \frac{\pi}{64}\int_0^2 (2x^4 - x^5)\,dx$$
$$= \frac{\pi}{64}\left[\frac{2}{5}x^5 - \frac{1}{6}x^6\right]_0^2$$
$$= \frac{\pi}{64}\left[\frac{64}{5} - \frac{64}{6} - 0\right] = \frac{\pi}{64}\left[\frac{32}{15}\right]$$
$$= \boxed{\frac{\pi}{30}\ m^3}$$

5. $$\frac{8x^3+13x}{(x^2+2)^2} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2}$$
$$8x^3+13x = (Ax+B)(x^2+2) + Cx + D$$
$$\equiv Ax^3 + Bx^2 + 2Ax + 2B + Cx + D$$

$A=8$ $B=0$ $2A+C=13$ $2B+D=0$

$16+C=13$ $D=0$

$C=-3$

$$\int \frac{8x^3+13x}{(x^2+2)^2}\,dx = \int \frac{8}{x^2+2}\,dx - \int \frac{3x}{(x^2+2)^2}\,dx$$

$u = x^2+2$
$du = 2x\,dx$

$$= \frac{8}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + \frac{3}{2}\int \frac{1}{x^2+2} + C$$
$$= \boxed{\frac{8}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + \frac{3}{2(x^2+2)} + C}$$

6. $\ln y = x \ln(\sin x)$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} x \ln(\sin x)$$

$$= \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}}$$

$$= \lim_{x\to 0^+} \frac{\frac{\cos x}{\sin x}}{-1/x^2}$$

$$= \lim_{x\to 0^+} \frac{-x^2 \cos x}{\sin x}$$

$$= \lim_{x\to 0^+} \frac{x^2 \sin x - 2x\cos x}{\cos x}$$

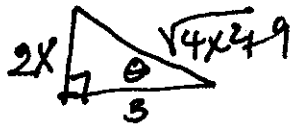
$$= 0$$

So, $\boxed{\lim_{x\to 0^+} (\sin x)^x = e^0 = 1}$

7. $\int \theta \sec\theta \tan\theta \, d\theta$ $\quad u=\theta \quad dv=\sec\theta\tan\theta\, d\theta$ $\quad du=d\theta \quad v=\sec\theta$

$$= \theta\sec\theta - \int \sec\theta \, d\theta$$

$$= \boxed{\theta\sec\theta - \ln|\sec\theta + \tan\theta| + C}$$



8. $\int \frac{1}{x\sqrt{4x^2+9}}\,dx$

$2x = 3\tan\theta$

$dx = \frac{3}{2}\sec^2\theta\,d\theta$

$$\int \frac{1}{\frac{3}{2}\tan\theta\sqrt{9\tan^2\theta+9}} \cdot \frac{3}{2}\sec^2\theta\,d\theta$$

$$= \frac{1}{3}\int \frac{\sec^2\theta}{\tan\theta\sec\theta}\,d\theta$$

$$= \frac{1}{3}\int \frac{\sec\theta}{\tan\theta}\,d\theta = \frac{1}{3}\int \csc\theta\,d\theta$$

$$= -\frac{1}{3}\ln|\csc\theta + \cot\theta| + C$$

$$= \boxed{-\frac{1}{3}\ln\left|\frac{\sqrt{4x^2+9}}{2x} + \frac{3}{2x}\right| + C}$$

9. $\int \sin^2(3x)\,dx = \frac{1}{2}\int (1-\cos 6x)\,dx$

$$= \boxed{\frac{1}{2}x - \frac{1}{12}\sin 6x + C}$$

10. $\int \sec^3 x\,dx = \int \sec x(1+\tan^2 x)\,dx$

$= \int \sec x\,dx + \int \tan x \sec x \tan x\,dx$

$u = \tan x \quad dv = \sec x\tan x\,dx$

$du = \sec^2 x \quad v = \sec x$

$\int \sec^3 x\,dx = \ln|\sec x + \tan x| + \sec x\tan x - \int \sec^3 x\,dx$

$2\int \sec^3 x\,dx = \ln|\sec x + \tan x| + \sec x\tan x$

$$\int \sec^3 x\,dx = \boxed{\frac{1}{2}\ln|\sec x + \tan x| + \frac{1}{2}\sec x\tan x + C}$$