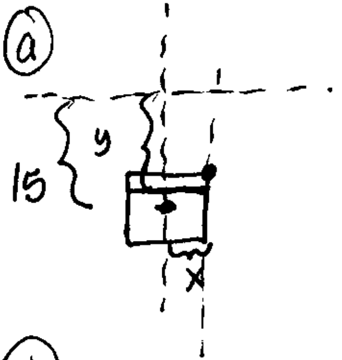
960

1. ⓐ



$$\boxed{F = 64\int_{-15.5}^{-14.5} (-y)(1)\,dy}$$

$$-32y^2\Big]_{-15.5}^{-14.5} = 960$$

ⓑ

$F = kd$

$250 = k(30)$

$k = \frac{25}{3}$

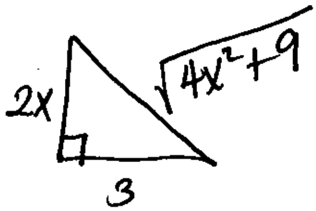
$$\boxed{W = \int_{20}^{50} \frac{25}{3}x\,dx}$$

2.

$u = a\tan\theta$

$2x = 3\tan\theta$

$2\,dx = 3\sec^2\theta\,d\theta$



$$\int \frac{\sqrt{4x^2+9}}{x^4}\,dx = \int \frac{\sqrt{9\tan^2\theta+9}}{(\frac{3}{2}\tan\theta)^4}\cdot\frac{3}{2}\sec^2\theta\,d\theta$$

$$= \frac{9}{2}\cdot\frac{2^4}{3^4}\int\frac{\sec^3\theta}{\tan^4\theta}\,d\theta$$

$$= \frac{2^3}{9}\int\frac{1}{\cos^3\theta}\cdot\frac{\cos^4\theta}{\sin^4\theta}\,d\theta$$

$$= \frac{8}{9}\int\frac{\cos\theta}{\sin^4\theta}\,d\theta$$

$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$= \frac{8}{9}\int u^{-4}\,d\theta$$

$$= \frac{8}{9}\left(-\frac{1}{3}\right)\left(\frac{1}{\sin^3\theta}\right) + C$$

$$= -\frac{8}{\ldots}\left(\csc^3\theta\right) + C = \boxed{\frac{-8}{\ldots}\left(\frac{\sqrt{4x^2+9}}{\ldots}\right)^3 + C}$$

3. $\int \frac{x^2+2x}{x^2(x-1)+(x-1)}\,dx$

$$\frac{x^2+2x}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$x^2+2x = A(x^2+1) + (Bx+C)(x-1)$$
$$x^2+2x = Ax^2+A+Bx^2-Bx+Cx-C$$

$1 = A+B$
$2 = -B+C$
$A-C=0$

$3 = A+C$
$\underline{0 = A-C}$
$3 = 2A$
$A = 3/2$

$C = 3/2$

$1 = 3/2 + B$
$B = -1/2$

$$\int \frac{3/2}{x-1}\,dx + \int \frac{-1/2\,x + 3/2}{x^2+1}\,dx$$

$$\frac{3}{2}\ln|x-1| - \frac{1}{2}\int \frac{x}{x^2+1}\,dx + \frac{3}{2}\tan^{-1}x + C$$

$u = x^2+1$
$du = 2x\,dx$

$$\boxed{\frac{3}{2}\ln|x-1| - \frac{1}{4}\ln|x^2+1| + \frac{3}{2}\tan^{-1}x + C}$$

4. a) $\boxed{a_n = \frac{2^n}{(n-1)!}}$

b) $\lim_{n\to\infty} \frac{2n-1}{3n+5} = \frac{2}{3}$

$\boxed{\text{The sequence converges to } 2/3.}$

5. $\sum 3\left(\frac{4}{3}\right)^n$

$\boxed{\text{GST } r = 4/3 \text{ Diverges}}$

6. $\sum \left[4+(-1)^n\right]$

$\lim_{n\to\infty} \left[4+(-1)^n\right] = \text{DNE} \neq 0$

Diverges by n^{th} term test

7. $\sum \frac{n}{8n^3+6n^2-7}$

$\boxed{\text{Converges by LCT}}$

Compare to $\sum \frac{1}{n^2}$ → Conv., P-Series, $p=2$

$\lim_{n\to\infty} \left(\frac{n}{8n^3+6n^2-7} \cdot \frac{n^2}{1}\right) = \frac{1}{8} \begin{matrix} >0 \\ <\infty \end{matrix}$

8. $\lim_{n\to\infty} \left| \frac{3\cdot5\cdot7\cdot\cdots(2n+1)(2n+3)}{(n+1)!} \cdot \frac{n!}{3\cdot5\cdot7\cdots(2n+1)} \right|$

$= \lim_{n\to\infty} \left(\frac{2n+3}{n+1}\right) = 2$

Diverges

9. $\sum \frac{1}{n(\ln n)}$ $\boxed{\text{Diverges by IT}}$

$f(x) = \frac{1}{x(\ln x)}$ $\quad \int_2^\infty \frac{1}{x \ln x}\, dx$ $\quad u = \ln x$, $du = \frac{1}{x}\, dx$

f cont ✓ $(x > 1)$ $\quad = \lim_{b \to \infty} \int_2^b \frac{1}{u}\, du$

f pos ✓

f dec ✓ $\quad = \lim_{b \to \infty} \left[\ln|\ln x| \right]_2^b$

$$= \lim_{b \to \infty} \left[\ln|\ln b| - \ln|\ln 2| \right]$$

$$= \infty$$

10. $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1} \right)$ $\boxed{\text{Telescoping Series Converges to 1.}}$

$$S_n = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right) + \left(\frac{1}{n+1} - \frac{1}{n+2}\right)$$

$$S_n = 1 - \frac{1}{n+2}$$

$$\lim_{n \to \infty} \left(1 - \frac{1}{n+2}\right) = 1$$

11. $f(x) = \sqrt[3]{x} \to f(8) = 2$

$$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3(\sqrt[3]{x})^2} \to f'(8) = \frac{1}{3(4)} = \frac{1}{12}$$

$$f''(x) = -\frac{2}{9} x^{-5/3} = -\frac{2}{9(\sqrt[3]{x})^5} \to f''(8) = \frac{-2}{9(32)} = \frac{-1}{144}$$

$$f'''(x) = \frac{10}{27} x^{-8/3} = \frac{10}{27(\sqrt[3]{x})^8} \to f'''(8) = \frac{10}{27(2^8)} = \frac{5}{3456}$$

$$P_4(x) = 2 + \frac{1}{12}(x-8) - \frac{1}{144} \cdot \frac{1}{2}(x-8)^2 + \frac{5}{3456} \cdot \frac{1}{6}(x-8)^3$$