Calculus II Exam #3
November 14, 2002

Name Key

Show all your work neatly on this paper. Solutions without correct supporting work will not be accepted. Each problem is worth 10 points. You may omit one problem by clearly writing "OMIT" by the problem. If you do not omit a problem then I will omit the last one for you. You may work the problem you omit for up to 5 bonus points if you wish.

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

1. Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n3^n}$.

$$\lim_{n\to\infty}\left|\frac{(x+2)^{n+1}}{(n+1)3^{n+1}} \cdot \frac{n3^n}{(x+2)^n}\right| = |x+2| \lim_{n\to\infty}\left|\frac{n}{3(n+1)}\right|$$

$$= |x+2| \cdot \frac{1}{3} < 1$$

$$-3 < x+2 < 3$$

$$-5 < x < 1$$

Check Endpoints

$x=-5$: $\sum \frac{(-3)^n}{n3^n} = \sum \frac{(-1)^n}{n} \to$ Conv. by AST

$x=1$: $\sum \frac{3^n}{n3^n} = \sum \frac{1}{n} \to$ Div. P-series

$\boxed{\text{IOC: } [-5, 1)}$

2. Find a power series, centered at 2, for the function $f(x) = \frac{1}{3x-2}$.

-4 if not centered properly

$$\frac{1}{3(x-2)-2+6} = \frac{1}{4+3(x-2)} = \frac{1/4}{1-\frac{-3}{4}(x-2)}$$

$\boxed{\sum_{n=0}^{\infty} \frac{1}{4}\left[-\frac{3}{4}(x-2)\right]^n}$ $\boxed{\text{IOC: } (2/3, 10/3)}$

+2 Bonus for IOC

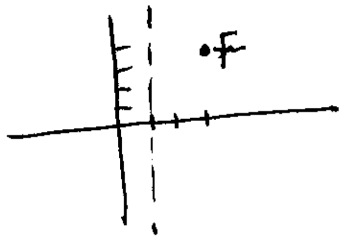
$$-1 < \frac{3}{4}(x-2) < 1 \qquad \frac{2}{3} < x < \frac{10}{3}$$

$$-\frac{4}{3} < x-2 < 4/3$$

3. Find the power series representation for $f(x) = e^{\sqrt{x}}$. Write the first 4 terms and the general term of the series.

$$\sum_{n=0}^{\infty} \frac{(\sqrt{x})^n}{n!} = 1 + \sqrt{x} + \frac{x}{2} + \frac{x^{3/2}}{6} + \cdots$$

4. Find an equation of the parabola with focus at (3,4) and directrix x=1.

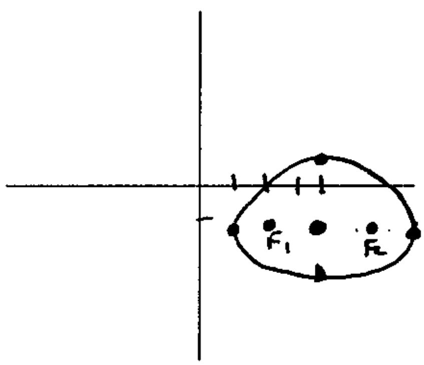


$V: (2,4) \quad p = 1$

$$\boxed{(y-4)^2 = 4(x-2)}$$

5. Find all that apply and sketch the graph: vertices, center, foci, directrix, asymptotes, lengths of minor and major axes.

$4x^2 + 9y^2 - 32x + 18y + 37 = 0$



$$4x^2 - 32x + 9y^2 + 18y = -37$$
$$4(x^2 - 8x + 16) + 9(y^2 + 2y + 1) = -37 + 64 + 9$$
$$4(x-4)^2 + 9(y+1)^2 = 36$$
$$\frac{(x-4)^2}{9} + \frac{(y+1)^2}{4} = 1$$

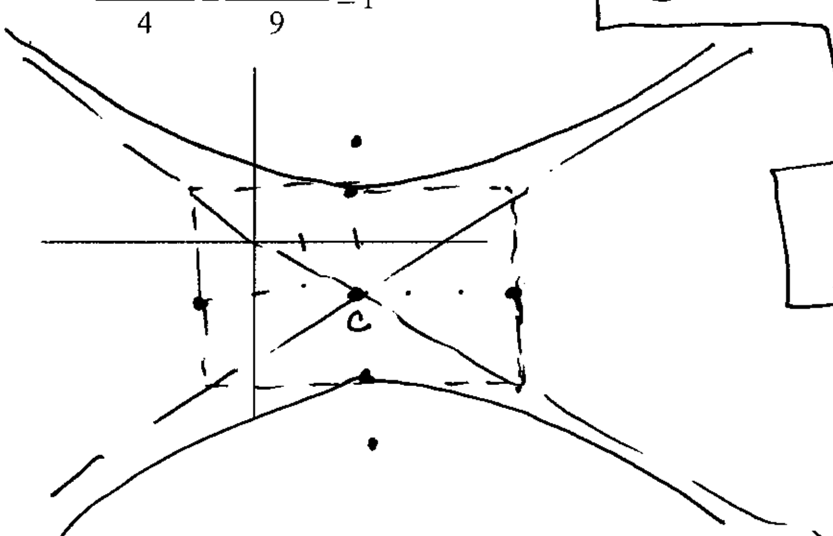
Center: $(4,-1)$
minor Axis = 4, Major Axis = 6
Vertices: $(1,-1)$, $(7,-1)$
Foci: $(4 \pm \sqrt{5}, -1)$

$c^2 = a^2 - b^2$
$c^2 = 9 - 4$
$c = \sqrt{5}$

$e = c/a = \sqrt{5}/3 \rightarrow$ +1 Bonus for eccentricity

6. Find all that apply and sketch the graph: vertices, center, foci, directrix, asymptotes, lengths of minor and major axes.

$$\frac{(y+1)^2}{4} - \frac{(x-2)^2}{9} = 1$$



Center: $(2,-1)$
Vertices: $(2,1)$, $(2,-3)$
Foci: $(2, -1 \pm \sqrt{13})$
Asymptotes: $y+1 = \pm \frac{2}{3}(x-2)$

$c^2 = a^2 + b^2$
$c^2 = 4 + 9 = 13$

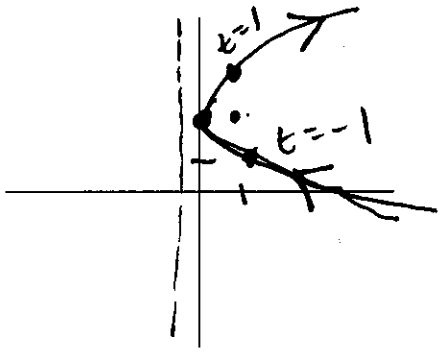
7. a. Find the corresponding rectangular equation for the curve represented by the parametric equations $x = t^2 + 2$ and $y = t^2 - 1$.

$t^2 = X - 2$

$y = -2 + x - 1$

$\boxed{y = X - 3}$

b. Sketch the curve represented by the parametric equations $x = t^2$ and $y = 2 + t$. Make sure you indicate the orientation of the curve and show how you arrived at your answer.



t	x	y
0	0	2
1	1	3
-1	1	1

$t = y - 2$

$x = (y-2)^2$

V: $(0, 2)$

$4p = 1$

$p = 1/4$

8. Find $\frac{d^2y}{dx^2}$ for the curve given by $x = \sqrt{t}$ and $y = (t-1)^3$.

$$\frac{dy}{dx} = \frac{3(t-1)^2}{\frac{1}{2}t^{-1/2}} = 6t^{1/2}(t-1)^2$$

$$\frac{d^2y}{dx^2} = \frac{6t^{1/2}\cdot 2(t-1) + 3t^{-1/2}(t-1)^2}{\frac{1}{2}t^{-1/2}} = \frac{12t(t-1) + 3(t-1)^2}{\frac{1}{2}\text{[redacted]}}$$

$$= \frac{24t^2 - 24t + 6t^2 - 12t + 6}{\text{[redacted]}} = \boxed{\frac{30t^2 - 36t + 6}{\text{[redacted]}}}$$

$\begin{array}{r} 34 \\ 19 \\ \hline 53 \end{array}$

9. Find the equation of the tangent line for the curve given by $x = 3t - 1$ and $y = t^2$ at the point where $t = 1$.

$$\frac{dy}{dx} = \left.\frac{2t}{3}\right|_{t=1} = 2/3$$

$(2, 1)$

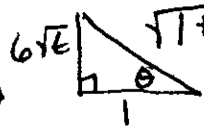
$\boxed{Y - 1 = \frac{2}{3}(X - 2)}$

10. Find the arc length for the curve given by $x = \sqrt{t}$ and $y = 3t - 1$ over the interval [0.1].

$$s = \int_0^1 \sqrt{\left(\tfrac{1}{2}t^{-1/2}\right)^2 + 3^2}\,dt = \int_0^1 \sqrt{\frac{1}{4t} + 9}\,dt$$

$$= \int_0^1 \sqrt{\frac{1+36t}{4t}}\,dt = \int_0^1 \frac{1}{2}\cdot\frac{1}{\sqrt{t}}\sqrt{1+36t}\,dt$$

$u = a\tan\theta$



$6\sqrt{t} = \tan\theta$

$3t^{-1/2}\,dt = \sec^2\theta\,d\theta$

$\frac{3}{\sqrt{t}}\,dt = \sec^2\theta\,d\theta$

$$= \frac{1}{2}\cdot\frac{1}{3}\int_{t=0}^{1} \sqrt{1+\tan^2\theta}\,\sec^2\theta\,d\theta$$

$$= \frac{1}{6}\int_{t=0}^{1} \sec^3\theta\,d\theta$$

$$= \frac{1}{12}\Big[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\Big]_{t=0}^{1}$$

$$= \frac{1}{12}\left[\frac{\sqrt{1+36t}}{1}\cdot\frac{6\sqrt{t}}{1} + \ln\left|\sqrt{1+36t} + 6\sqrt{t}\right|\right]_0^1$$

$$= \frac{1}{2}\sqrt{37}\sqrt{1} + \frac{1}{12}\ln|\sqrt{37}+6| - \frac{1}{12}[1(0) + \ln 1]$$

$$= \boxed{\frac{\sqrt{37}}{2} + \frac{\ln(6+\sqrt{37})}{12}}$$

$\int \sec^3\theta\,d\theta = \int \sec\theta\sec^2\theta\,d\theta$

$u = \sec\theta \qquad dv = \sec^2\theta\,d\theta$

$du = \sec\theta\tan\theta\,d\theta \qquad v = \tan\theta$

$\int \sec^3\theta\,d\theta = \sec\theta\tan\theta - \int \sec\theta\tan^2\theta\,d\theta$

$= \sec\theta\tan\theta - \int \sec\theta(-1+\sec^2\theta)\,d\theta$

$= \sec\theta\tan\theta + \int \sec\theta\,d\theta - \int \sec^3\theta\,d\theta$

$\int \sec^3\theta\,d\theta = \frac{1}{2}\big[\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|\big] + C$

11. Find the area of the surface generated by revolving the curve $x = 4\cos\theta,\ y = 4\sin\theta,\ 0 \le \theta \le \pi/2$ about the y-axis.

$$S = 2\pi\int_0^{\pi/2} 4\cos\theta\sqrt{(-4\sin\theta)^2 + (4\cos\theta)^2}\,d\theta$$

$$= 8\pi\int_0^{\pi/2} \cos\theta\sqrt{16(\sin^2\theta + \cos^2\theta)}\,d\theta$$

$$= 32\pi\int_0^{\pi/2} \cos\theta\,d\theta$$

$$= 32\pi\Big[\sin\theta\Big]_0^{\pi/2}$$

$$= 32\pi(1-0) = \boxed{32\pi}$$