Calculus II Quiz #3

September 30, 2008

Name ________________________________

Show all your work neatly and in numerical order on notebook paper. Solutions without correct supporting work will not earn credit.

1. Evaluate:
   a. $\lim_{x\to 0} \frac{\sin \pi x}{\sin 2\pi x}$
   b. $\lim_{x\to \infty} (1+\frac{2}{x})^x$

2. Evaluate: $\int_0^2 \frac{1}{4-x^2}\,dx$

3. Evaluate: $\int \frac{8x^3+13x}{(x^2+2)^2}\,dx$

4. Evaluate: $\int_0^\infty (x-1)e^{-x}\,dx$

Quiz #3 Key - Calculus II - Fall 2008

1. a) $\lim_{x\to 0} \frac{\sin \pi x}{\sin 2\pi x} = \lim_{x\to 0} \frac{\pi \cos \pi x}{2\pi \cos 2\pi x} = \frac{\pi}{2\pi} = \boxed{\frac{1}{2}}$

b) $y = \left(1+\frac{2}{x}\right)^x$

$\ln y = x \ln\left(1+\frac{2}{x}\right)$

$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} x \ln\left(\frac{x+2}{x}\right)$

$= \lim_{x\to\infty} \frac{\ln(1+2/x)}{\frac{1}{x}}$

$\overset{LH}{=} \lim_{x\to\infty} \frac{\left(\frac{x}{x+2}\right)\left(-\frac{2}{x^2}\right)}{-1/x^2}$

$= \lim_{x\to\infty} 2\left(\frac{x}{x+2}\right) = 2$

So,

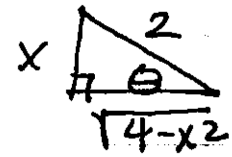
$\boxed{\lim_{x\to\infty} \left(1+\frac{2}{x}\right)^x = e^2}$

2. $\int_0^2 \frac{1}{4-x^2}\,dx$

$u = a\sin\theta$

$x = 2\sin\theta$

$dx = 2\cos\theta\,d\theta$



$= \int_0^2 \frac{1}{4-4\sin^2\theta}\cdot 2\cos\theta\,d\theta$

$= \frac{1}{2}\int_0^2 \sec\theta\,d\theta$

$= \frac{1}{2}\lim_{b\to 2^-}\Big[\ln|\sec\theta+\tan\theta|\Big]_{x=0}^{b}$

$= \frac{1}{2}\lim_{b\to 2^-}\left[\ln\left|\frac{2}{\sqrt{4-x^2}}+\frac{x}{\sqrt{4-x^2}}\right|\right]_0^{b}$

$= \frac{1}{2}\lim_{b\to 2^-}\left[\ln\left|\frac{2}{\sqrt{4-b^2}}+\frac{b}{\sqrt{4-b^2}}\right| - \ln\frac{2}{2} - 0\right]$

$= \frac{1}{2}\lim_{b\to 2^-}\ln\left|\underbrace{\frac{2+b}{\sqrt{4-b^2}}}_{\text{Form } \frac{4}{0} \to \text{ Approaches } \infty}\right| - 0$

$= \infty$

$\boxed{\text{The integral } \int_0^2 \frac{1}{4-x^2}\,dx \text{ \underline{diverges}}}$

3) $\int \frac{8x^3+13x}{(x^2+2)^2}\,dx$

$$\frac{8x^3+13x}{(x^2+2)^2} = \frac{Ax+B}{x^2+2} + \frac{Cx+D}{(x^2+2)^2}$$

$$8x^3+13x = (Ax+B)(x^2+2) + Cx+D)$$
$$= Ax^3+2Ax+Bx^2+2B+Cx+D$$
$$= Ax^3+Bx^2+(2A+C)x+2B+D$$

$A=8$ $\quad$ $B=0$ $\quad$ $2A+C=13$ $\quad$ $2B+D=0$

$16+C=13$ $\quad$ $D=0$

$C=-3$

So,

$$\int \frac{8}{x^2+2}\,dx + \int \frac{-3x}{(x^2+2)^2}\,dx$$

$u=x^2+2$

$du=2x\,dx$

$$\frac{8}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) - \frac{3}{2}\int u^{-2}\,du$$

$$\boxed{\frac{8}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{3}{2(x^2+2)} + C}$$

4. $\int_0^\infty (x-1)e^{-x}\,dx$

$u = x-1 \quad dv = e^{-x}dx$
$du = dx \quad v = -e^{-x}$

$$\lim_{b\to\infty}\left[-(x-1)e^{-x} + \int e^{-x}\,dx\right]_0^b$$

$$= \lim_{b\to\infty}\left[-(x-1)e^{-x} - e^{-x}\right]_0^b$$

$$= \lim_{b\to\infty}\left[\frac{-(b-1)}{e^b} - \frac{1}{e^b} + (-1) + 1\right]$$

$$= \lim_{b\to\infty}\left[\frac{-b}{e^b}\right]$$

$$\overset{LH}{=} \lim_{b\to\infty}\left[\frac{-1}{e^b}\right] = 0$$

$\boxed{\text{Converges to } 0}$