Discrete Math Exam #1
October 2, 2002

Name ____________________________________

1. Fill in the truth table. You may add any columns you wish but write your answer in the one provided. (8 points)

p	q	$p \to \sim(p \vee q)$				
T	T					
T	F					
F	T					
F	F					

2. Write the negation of the following statements. (4 points each)

   a. If Sally is sixteen then she has a driver's license.

   b. Everyone who is sixteen has a driver's license.

3. Determine whether the following argument is valid or invalid. You must either show a truth table or explain your answer. (8 points)

   If 2=3, then I ate my hat.
   I ate my hat.
   Therefore, 2=3.

4. Let U = {a,b,c,d,e}, A = {a,c}, B = {a,b,d}, C = {b,e}. Find: (4 points each)

   a. $A \cap \overline{B \cup C}$ = ______________________

   b. $\overline{A} - B$ = ________________________

   c. $A \times B$ = __________________________

5. Consider the relation R on the set {1,2,3,4,5} defined by the rule $(x, y) \in$ R if $x + y \leq 6$. (4 points each)

   a. List the elements of R.

   b. Is R reflexive? Why or why not?

   c. Is R symmetric? Why or why not?

   d. Is R transitive? Why or why not?

   e. Is R a partial order? Why or why not?

6. True or False. (3 points each)

_______ a. {{1,2}, {3}, {4}} is an equivalence relation on {1,2,3,4}.

_______ b. {1,2} $\in$ {1,2,3,4}

_______ c. If R is a relation, R^{-1} always exists.

7. Give an example of a function $f : X \to Y$ that is: (Make sure you define X and Y. You may use either a formula or a diagram to represent your functions.) (4 points each)

   a. onto

   b. one-to-one

   c. onto but not one-to-one

8. True or False. If false, you must give a counterexample (an example to show it is false). (4 points each)

_______ a. For every y, for some x, $x^2 < y + 1$. The domain of discourse is the set of real numbers.

_______ b. $\overline{X \cap Y} \subseteq X$ for all sets X and Y.

_______ c. If R and S are antisymmetric, then R $\cup$ S is antisymmetric.

_______ d. If f^{-1} is a function, then f is a bijection.

9. Use mathematical induction to prove: $1(2)+2(3)+3(4)+\cdots+n(n+1)=\dfrac{n(n+1)(n+2)}{3}$ (10 points)