

Show all your work on this paper. Solutions without correct supporting work will not be accepted.

1. Use the Euclidean algorithm to find the greatest common divisor of 2475 and 32670. (5 points)

2. How many eight-bit strings begin and end with 1? (5 points)

3. A club consists of 5 women and 7 men. Simplify your answers. (5 points each)
 - a. In how many ways can a president, vice-president, and secretary be selected if there are no restrictions?

 - b. In how many ways can a president, vice-president, and secretary be selected if a woman must be president and a man must be the secretary?

 - c. In how many ways can a committee of four people be selected to go to a seminar if there must be 2 women and 2 men on the committee?

4. Find the number of (unordered) five-card poker hands, selected from an ordinary 52-card deck, having the properties indicated. Do not simplify your answers. (5 points each)
 - a. Containing all four 8's.

 - b. Containing cards of exactly 2 suits.

 - c. What is the probability of being dealt a hand that contains exactly 2 suits?

5. Two cards are selected at random from an ordinary deck without replacement. Write your answer as a fraction. (5 points each)
 - a. What is the probability of selecting an ace and a king, in that order?

 - b. What is the probability of selecting an ace and a king if order does not matter?

6. Determine the number of strings that can be formed by ordering the letters in ARKANSAS. (5 points)
7. In how many ways can 12 distinct books be divided among four students if each student gets three books? (5 pts)
8. Ann has a collection of text books. Since she has several copies of the same history, math, and economics texts she plans to give 10 books to a book drive. (She has at least 10 copies of each text.) (5 points each)
- a. In how many ways can she select 10 books to give away?
- b. In how many ways can she select 10 books to give away if she wants to make sure she gives at least one copy of each of the three texts?
9. Find the permutation that will follow 625431 in lexicographic order. (5 points)
10. Is the relation R defined on $\{1,2,3\}$ by the rule (x,y) is in R if $x+y \leq 6$ an equivalence relation? You must justify your answer by showing all your work. You may ask me the definition of an equivalence relation for a deduction of 3 points. (8 points)
11. a. What is the difference between a relation and a function? (3 points)
- b. Give an example of a function that is onto. (4 points)

12. Use mathematical induction to show $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$. (8 points)

BONUS

Write three different statements that are equivalent to the statement below. (“If I study, I will pass the test” and “I will pass the test if I study” don’t count.)

If I study then I will pass the test.