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Formula for the Future (Accumulated) Value of an Ordinary Annuity

For an ordinary annuity, payment is made at the **end** of the period and the interest is compounded at the same time the payment is made.

$$A = p \cdot \left[\frac{\left(1+\frac{r}{m}\right)^{m \cdot t} - 1}{\frac{r}{m}} \right] = p \cdot \left[\frac{(1+i)^n}{i} \right]$$

A is the accumulated or future value of the annuity after t years
p is the payment made at the end of each compounding period
r is the annual compound interest rate, compounded m times per year, in decimal form
m is the number of compounding periods (payments) per year
t is the number of years

$i = \frac{r}{m}$ = interest rate per period
$n = m \cdot t$ = total number of interest periods (payments)

Example:

Find the future value of an annuity if $300 is deposited at the end of every month for two years at 4.5% annual interest.

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Formula for the Future (Accumulated) Value of an Ordinary Annuity with a Starting Balance

$$A = P\cdot\left(1+\frac{r}{m}\right)^{(m\cdot t)} + p\cdot\left(\frac{\left(\left(1+\frac{r}{m}\right)^{(m\cdot t)}-1\right)}{\left(\frac{r}{m}\right)}\right) = P\cdot(1+i)^{n} + p\cdot\left[\frac{(1+i)^{n}-1}{i}\right]$$

A is the future (accumulated) value of the annuity after t years
P is the initial amount in the account.
p is the payment made at the <u>end</u> of each compounding period
r is the annual compound interest rate, compounded m times per year, in decimal form
m is the number of compounding periods (payments) per year
t is the number of years

$i = \frac{r}{m}$ = interest rate per period

$n = m\cdot t$ = total number of interest periods (payments)

P. 283, #14

- Timothy has $10,000 in an account now and plans to add $1000 to the account at the end of each quarter from now on. If the account is assumed to pay 6% compounded quarterly, how much will be in it at the end of eight years?

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Sinking Funds

A sinking fund is an annuity established to accumulate a specified amount of money by a specified date. To find the payment necessary to accumulate the specified amount, we solve the annuity formula for p.

Formula for a Payment into an Ordinary Annuity Sinking Fund

$$p = A \cdot \frac{\left(\frac{r}{m}\right)}{\left(\left(1+\frac{r}{m}\right)^{(m \cdot t)} - 1\right)} = A \cdot \frac{i}{\left((1+i)^n - 1\right)}$$

Payments are made at the end of the period.

A is the desired future (accumulated) value of the annuity after t years
p is the payment made at the end of each compounding period
r is the annual compound interest rate, compounded m times per year, in decimal form
m is the number of compounding periods (payments) per year
t is the number of years

$i = \frac{r}{m}$ = interest rate per period

$n = m \cdot t$ = total number of interest periods (payments)

P. 284, #22

- If you are 30 years of age now and if you plan to retire at age 60 with an IRA account having a total accumulated value of $300,000, how much would you have to invest at the end of each year if the account paid 6% compounded annually?

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Formula for the Present Value of an Ordinary Annuity

$$P = p \cdot \left[\frac{\left(1 - \left(1 + \frac{r}{m} \right)^{-(m \cdot t)} \right)}{\left(\frac{r}{m} \right)} \right] = p \cdot \left[\frac{1 - (1+i)^{-n}}{i} \right]$$

Payments are made at the end of the perioc

P is the initial amount in the account.
p is the payment made at the end of each compounding period

r is the annual compound interest rate, compounded m times per year, in decimal form
m is the number of compounding periods (payments) per year
t is the number of years

$i = \frac{r}{m}$ = interest rate per period

$n = m \cdot t$ = total number of interest periods (payments)

P. 284, #24

- The Brewsters are saving for their daughter's college days. They would like to be able to withdraw $800 each month from their account for five years once their daughter starts college. Assuming that their account will earn interest at the rate of 9% compounded monthly, what sum of money should the Brewsters have in the account when their daughter starts college?