

Section 10.4—Continuous Random Variables and the Normal Distribution

Terms

Continuous Random Variables—Random variables whose values can be any number within a specified interval.

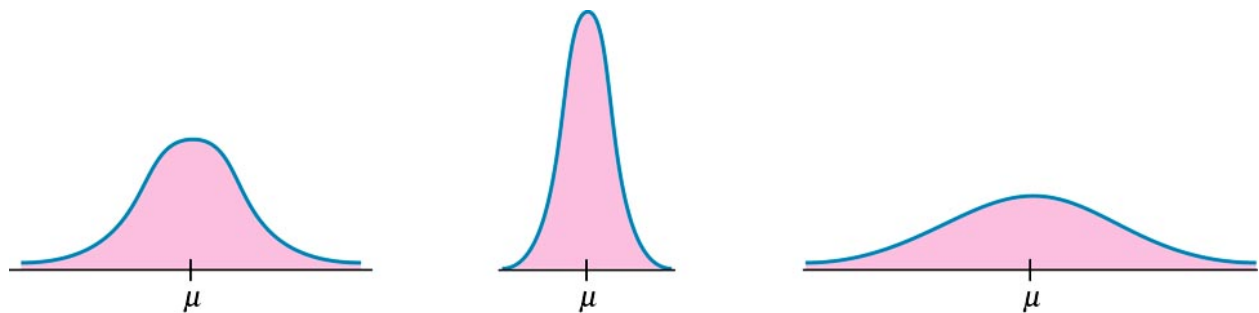
Examples include: fuel mileage of an automobile

Actual weight of a box of cereal

Weight of a newborn

How do we handle such data? Find the sample mean and standard deviation (to estimate the same for the population). Construct a corresponding relative frequency histogram and use that to draw a **probability distribution curve**. Then we try to match the shape of the sketched curve with the graph of one of several known probability density functions. In this section we will concentrate on the Normal Probability Distribution or Bell Curve.

There is not just one normal curve, but a different one for each value of the mean and standard deviation.



Three normal distributions

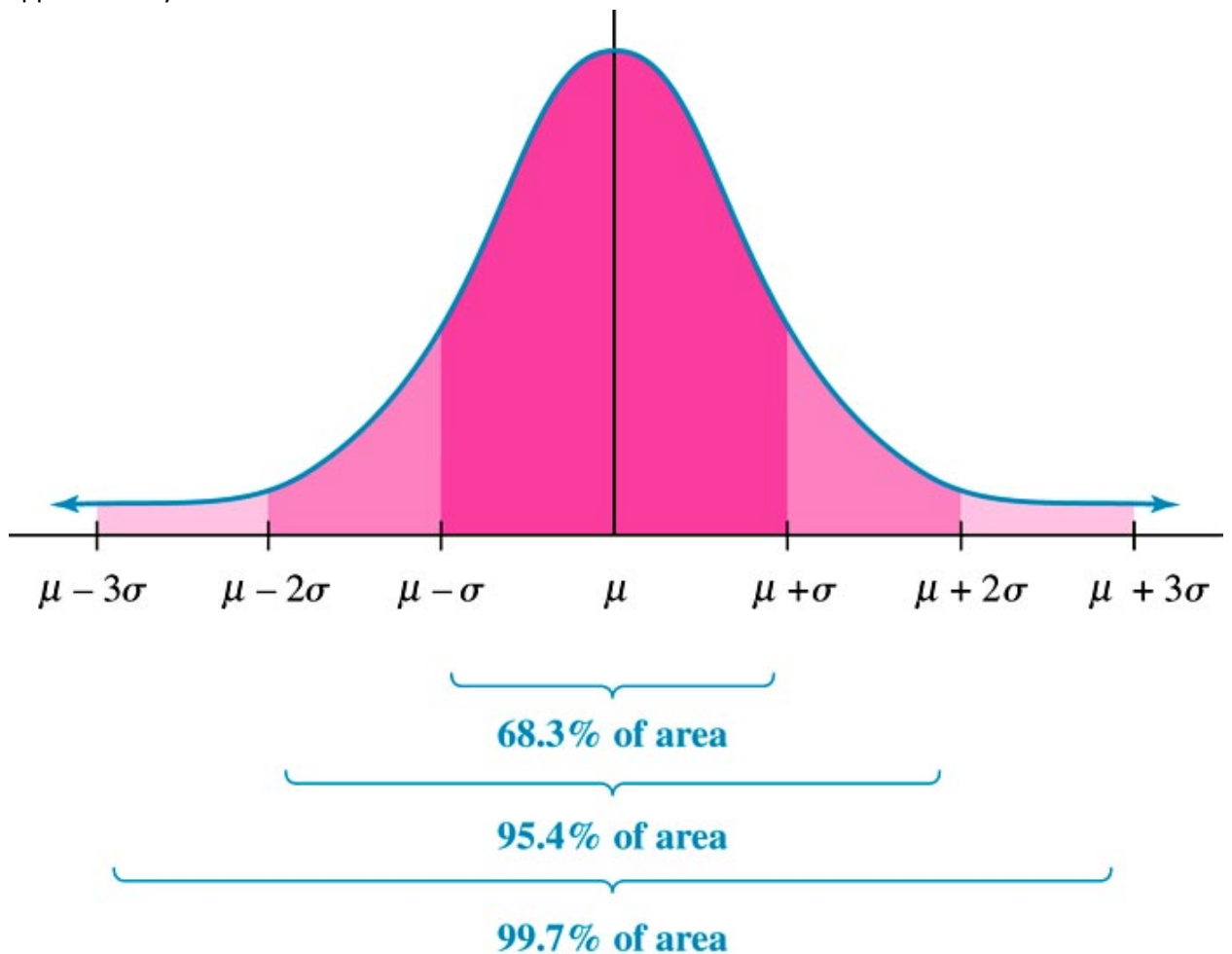
Even though different normal curves have different graphs, they do share several properties:

- 1) The highest point on the graph occurs at the mean
- 2) The graph is always symmetric about the vertical line through the mean.
- 3) The graph never touches the x-axis, although it gets infinitely close on both sides
- 4) The area below the curve and above the x-axis is always equal to 1
- 5) The area below the curve to the left of the mean = .5 and the area below the curve to the right of the mean is .5

All normally distributed random variables obey the Empirical Rule which states:

For a normally distributed random variable with mean μ and standard deviation σ ,

- 1) Approximately 68% of the area below the curve lies within one standard deviation of the mean.
Or, in another context. The probability that a data point lies within one standard deviation of the mean is approximately 68%.
- 2) Approximately 95% of the area under the curve lies within two standard deviations of the mean
- 3) Approximately 99.7% of the area lies within three standard deviations of the mean.



4)

Example: Suppose that the weights of boxes of cereal are normally distributed with a mean of 283 grams and a standard deviation of 4 grams. Find the following:

- 1) The coordinate of the point of the x-axis that lies 2 standard deviations below the mean.
- 2) The coordinate of the point on the x-axis that lies 1.6 standard deviations above the mean.
- 3) Find the approximate percentage of area under the normal curve between $x = 279$ and $x = 287$
- 4) Find the approximate percentage of area under the normal to the right of $x = 291$

For each of the above questions, a probability can be associated with the area under the normal curve. This area (or probability) is dependent on how far away from the mean the value of the random variable is.

Stated differently 3) above could have said: What is the probability that a box of cereal weighs between 279 and 283 grams?

To utilize the standard normal curve to find probabilities we first have to determine how many standard deviations a particular value lies away from the mean. We do that by calculating what is known as a z-score.

z-score- If X is a normally distributed random variable with mean μ and standard deviation σ , then the z-score for a data value x is given by $z = \frac{x-\mu}{\sigma}$. In other words, z is a measure of how many standard deviations x lies away from the mean.

Example: For the cereal boxes in the previous example, what is the z-score for an x-value of 285? 276?

To calculate probabilities for a normal distribution:

- 1) Compute the pertinent z-scores**
- 2) Use a table to find the corresponding probabilities. Keep in mind that the table in the appendix of our text book gives the percentage of area (or probability) to the left of (or less than) a given z-score. If we are asked for something else, such as probability to the right of or between two scores we will have to do more.**

Example: Suppose that the weights of boxes of cereal are normally distributed with a mean of 283 grams and a standard deviation of 4 grams. Find the following:

- a) The probability that a box weighs less than 280 grams
- b) The probability that a box weighs more than 288 grams
- c) The probability that a box weighs between 276 and 289 grams

#52 Hunt's department store has 50,000 credit card holders. During the month of January, the average billing to credit card holders (rounded to the nearest dollar) was \$85, with a standard deviation of \$12. Assuming that these billings are normally distributed,

- a) How many billings were for less than \$75
- b) How many billings were for more than \$100

#59 Suppose the weights of domestic short hair cats is normally distributed with a mean of 10 lbs, and a standard deviation of 2 lbs.

- a) What is the probability that a domestic short hair cat weighs between 7 and 13 lbs?
- b) Ender, a domestic short hair cat, weighs 19 lbs. What percentage of all domestic short hair cats weigh as much or more than Ender?