Optimizing Functions of Several Variables

The point (a,b) is a critical point of $f(x, y)$ if $f_x(a,b) = 0$ and $f_y(a,b) = 0$

To find the maxima, minima, and saddle points of a function $f(x, y)$ we first find the critical points as follows:

1) Set the partial derivatives equal to zero
2) Solve the resulting system of equations

Next, we apply the following test (called the D-test)

If (a,b) is a critical point of the function $f(x, y)$, then for D defined by : $D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a) f has a relative maximum at (a,b) if D>0 and $f_{xx}(a,b) < 0$

b) f has a relative minimum at (a,b) if D>0 and $f_{xx}(a,b) > 0$

c) f has a saddle point at (a,b) if D<0

Example 1 (#8 in 7.3)

Find the relative extreme values of each function $f(x, y) = 5xy - 2x^2 - 3y^2 + 5x - 7y + 10$

Example 2 (#16, 7.3)

Find the relative extreme values of $x^3 - y^2 - 3x + 6y$

Example 3 (# 24, 7.3)

In a laboratory test the combined antibiotic effect of x milligrams of medicine A and y milligrams of medicine B is given by the function $f(x, y) = xy - 2x^2 - y^2 + 110x + 60y$ (for $0 \le x \le 55$, $0 \le y \le 60$). Find the amounts of the two medicines that maximize the antibiotic effect.