Section 4.1 – The Law of Sines

Objective: In this section you will learn how to use the Law of Sines to solve oblique triangles and to solve application problems.

I. Introduction

The Law of Sines

If α, β, and γ are the measures of the angles of a triangle, and a, b, and c are the lengths of the sides opposite these angles, then

To solve an oblique triangle, you need to know the measure of at least one side and any two other parts of the triangle. Three cases that can be solved using the Law of Sines are:

(1)

(2)

(3)

All triangles in this chapter will be labeled as shown below.

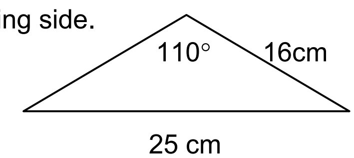
Example 1: For a triangle labeled in the standard way, $\alpha = 31.6°$, $\gamma = 42.9°$, and a = 10.4 meters. Find the missing parts.

II. The Ambiguous Case (SSA)

If two sides and one opposite angle of an oblique triangle are given, _______________ possible situations can occur, which are:

Example 2: For a triangle having $\alpha = 25°$, b=54 feet, and a = 26 feet, find the missing parts.

Example 3: For the triangle shown, Find the length of the missing side.



III. Applications of the Law of Sines

Example 4: Coast Guard Station Able is located 150 miles due south of Station Baker. A ship at sea sends an SOS call that is received by each station. The call to Station Able indicates that the ship is located N55°E; the call to Station Baker indicates that the ship is located S60°E.

(a) How far is each station from the ship?

(b) If a helicopter capable of flying 200 miles per hour is dispatched from the nearest station to the ship, how long will it take to reach the ship?

Example 5: The famous Leaning Tower of Pisa was originally 184.5 feet high. At a distance of 123 feet from the base of the tower in the direction of the lean, the angle of elevation to the top of the tower is found to be 60°. Find the angle the tower makes with the ground in the direction of the lean. Also, find the perpendicular distance from the top of the tower to the ground.

Section 4.2 – The Law of Cosines

Objective: In this section you will learn to use the Law of Cosines to solve oblique triangles and applications. You will also learn to find the area of oblique triangles.

I. Introduction

The Law of Cosines

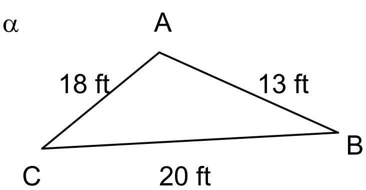
If α, β, and γ are the measures of the angles of a triangle, and a, b, and c are the lengths of the sides opposite these angles, then

(1)

(2)

(3)

Example 1: Using the triangle shown at the right, find angle α



When given the lengths of all three sides of a triangle and asked to find all three angles, always find the longest side first. Then you can use the Law of Sines to find a second angle without having to worry about running into the ambiguous case.

Example 2: For a triangle labeled in the usual way and given that α=62°, b=26 ft, and c=19 ft, find a.

II. Applications of the Law of Cosines

Example 3: A developer has a triangular lot at the intersection of two streets. The streets meet at an angle of 72°, and the lot has 300 feet of frontage along one street and 416 feet of frontage along the other street. Find the length of the third side of the lot.

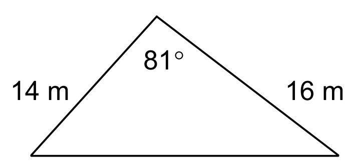
III. Area of Oblique Triangles

We all know the formula to find the area of a triangle, $A = \frac{1}{2}bh$. This is fine as long as you know the base and the height. The reality is often this information is not readily available.

If you know the measure of two sides of a triangle and the included angle then use the following formula to obtain the area:

> The area K of a triangle is one-half the product of the lengths of any two sides and the sine of the included angle. Thus,
>
> $$K = \frac{1}{2}bc\sin\alpha$$
>
> $$K = \frac{1}{2}ab\sin\gamma$$
>
> $$K = \frac{1}{2}ac\sin\beta$$

Example 4: Find the area of the triangle.



If you know the lengths of all the sides of a triangle, but not the angles, use Heron's Formula to find the area:

Heron's Area Formula states that given any triangle with sides of length a, b, and c, the area of the triangle is:

Area = $\sqrt{\qquad\qquad\qquad\qquad\qquad}$ where s = ______________________________

Example 5: Find the area of the triangle having sides of length 14 cm, 21 cm, and 27 cm.

Section 4.3 - Vectors

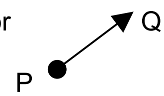
Objective: In this section you will learn to write the component forms of vectors, perform basic vector operations, and find the direction angle of vectors. You will also learn to find the dot product of two vectors and find the angle between two vectors.

I. Introduction

A scalar is a ________________________ ______________________. What are some things that we can measure with scalars?

Sometimes we need to measure things that include both a *magnitude* and a *direction*. This is when we need vectors. Examples:

A **vector** is a __________________________ ______________________ ________________________. The length of the line segment is the ___________________________ of the vector, and the direction of the vector is measured by an angle.

For the vector 

, P is the ________________________ point and Q is the _________________________ point.

If we label the above vector as vector **v**, we can write it as **v**, **PQ**, $\vec{v}$, or $\overrightarrow{PQ}$.

The **magnitude** of the directed line segment $\overrightarrow{PQ}$**,** denoted by ______________, is its __________________________.

Equivalent vectors have the same ________________________ and ______________________________. Thus we can place our vectors in standard position on our coordinate system.

II. Component Form of a Vector

A vector whose initial point is a the origin (0,0) an be uniquely represented by the coordinates of its terminal point (v_1, v_2).

This is the _____________________ form of vector **v**, written **v** = $\langle v_1, v_2 \rangle$, where v_1 and v_2 are the

____________________________ of **v**.

The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is

$\overrightarrow{PQ} =$ _____________________________ = __________________ = **v** .

The **magnitude** (or ______________________) of **v** is:

$\|v\| = \sqrt{\qquad\qquad\qquad} = \sqrt{\qquad\qquad}$

Example 1: Find the component form and magnitude of the vector **v** that has (1,7) as its initial point and (4,3) as its terminal point.

III. Vector Operations

In operations with vectors, numbers are usually referred to as ____________________________. Geometrically, the product of a vector **v** and a scalar *k* changes the _______________________of the vector, but not the direction.

Examples:

If *k* is positive, *k***v** has the ____________________ direction as **v**, and if *k* is negative, *k***v** has the ___________________ direction.

To add two vectors geometrically place the initial points at the origin and then complete the ________________________________. The sum of the two vectors is called the ___________________________________.

Example:

For component addition let **u** = $\langle u_1, u_2 \rangle$ and **v** = $\langle v_1, v_2 \rangle$ be vectors and let *k* be a scalar (a real number). Then the sum of **u** and **v** is the vector:

u + **v** =___________________________

and the scalar multiple of *k* times **u** is the vector:

*k***u** = ____________________________

Example 2: Let **u** = $\langle 1,6 \rangle$ and **v** = $\langle 4.2 \rangle$. Find: (a) 3**u** (b) **u** + **v**

IV. Unit Vectors

A unit vector is a vector whose magnitude is ______________.

To find a unit vector **u** that has the same direction as a given nonzero vector **v**, ____________________ each component of **v** by the _______________________________ of **v**.

Example 3: Find a unit vector in the direction of **v** = $\langle -8,6 \rangle$.

The standard unit vectors are the vectors *i* and *j* defined by *i*= _____________ and *j* = _______________.

Let v = $\langle v_1, v_2 \rangle$. Then the standard unit vectors can be used to represent **v** as **v** = _________________________, where the scalar v_1 is called the ______________________ component and the scalar v_2 is called the ____________________ component. The vector sum $v_1 i + v_2 j$ is called a ______________________ combination of the standard unit vectors.

Example 4: Let **v** = $\langle -5,3 \rangle$. Write **v** as a linear combination of the standard unit vectors.

Example 5: Let **v** = 3**i** – 4**j** and **w** = 2**i** +9**j.** Find **v** + **w**.

V. Direction Angles

The direction angle of a vector is the smallest positive angle the vector makes with the positive x-axis.

If **u** is a unit vector and θ is its direction angle, the terminal point of **u** lies on the unit circle and

u = $\langle x, y \rangle$ = ____________________________ .

Example 6: Let **v** = -4**i** + 5**j**. Find the direction angle for v.

VI. Applications of Vectors

Example 7: A plane is flying at an airspeed of 340 mph at a heading of 124°. A wind of 45 mph is blowing from the west. Find the ground speed of the plane. Also find the course of the plane.

VII. The Dot Product

The **dot product** of **u** = $\langle u_1, u_2 \rangle$ and **v** = $\langle v_1, v_2 \rangle$ is ____________________________________ . This product yields a ____________________________.

Note the properties of the dot product that are given on p. 247 in your text.

Example 8: Find the dot product: $\langle 5, -4 \rangle \bullet \langle 9, -2 \rangle$

VIII. **The Angle Between Two Vectors**

If θ is the smallest positive angle between two nonzero vectors **u** and **v**, then θ can be determined from

$\cos\theta =$ ________________________ .

Example 9: Find the angle between **v** = $\langle 5,-4 \rangle$ and **w** = $\langle 9,-2 \rangle$.

An alternative way to calculate the dot product between two vectors **u** and **v**, given the angle θ between them, is

v • w = ______________________________________ .

Two vectors are **orthogonal** (perpendicular) if _____________________________________ .

IX. Finding Vector Components

If **v** and **w** are two nonzero vectors and α is the smallest positive angle between **v** and **w,** then the scalar projection of **v** on **w**, $\text{proj}_{\mathbf{w}}\mathbf{v}$, is given by $\text{proj}_{\mathbf{w}}\mathbf{v}$ **=** ____________________ **=** ________________________**.**

Consider the diagram of the projection**:**

X. Work

The work done by a constant force **F** applied along a displacement **s** is given by either of the following:

1.

2.

Example 10: A 100-lb force is pulling a sled loaded with bricks that weighs 400 pounds. The force is at an angle of 42° with the displacement. Find the work done in moving the sled 25 feet.