Section 5.1 – Complex Numbers

I. The Imaginary Unit

Mathematicians created an expanded system of numbers using the **imaginary unit *i***, defined as *i* = ________________.

By definition, i^2 = ________________.

Each complex number can be written in the **standard form** ______________________, where the number __________ is called the **real part** of the complex number, and the number ____________ is called the **imaginary part** of the complex number.

Note that the set of real numbers is a subset of the set of complex numbers.

II. Operations with Complex Numbers

To add or subtract two complex numbers, …

Example 1: Perform the operations: $(5-6i)-(3-2i)+4i$

To multiply two complex numbers, use the ________________ method.

Example 2: Multiply: $(5-6i)(3-2i)$

III. Complex Conjugates and Division

The conjugate of $a+bi$ is _________________________ .

The product of a pair of complex conjugates is a ______________________ number.

To find the quotient $\dfrac{a+bi}{c+di}$ where c and d are not both zero,…

Example 3: Write in standard form. $\frac{1+i}{2-i}$

IV. Complex Solutions of Quadratic Equations

If a is a positive number, the **principal square root** of the negative number $-a$ is defined as ___________________.

To avoid problems with multiplying square roots of negative numbers, be sure to pull the _____ out of the radical before multiplying.

Example 4: Perform the operation and write the result in standard form. $\left(5-\sqrt{-4}\right)^2$

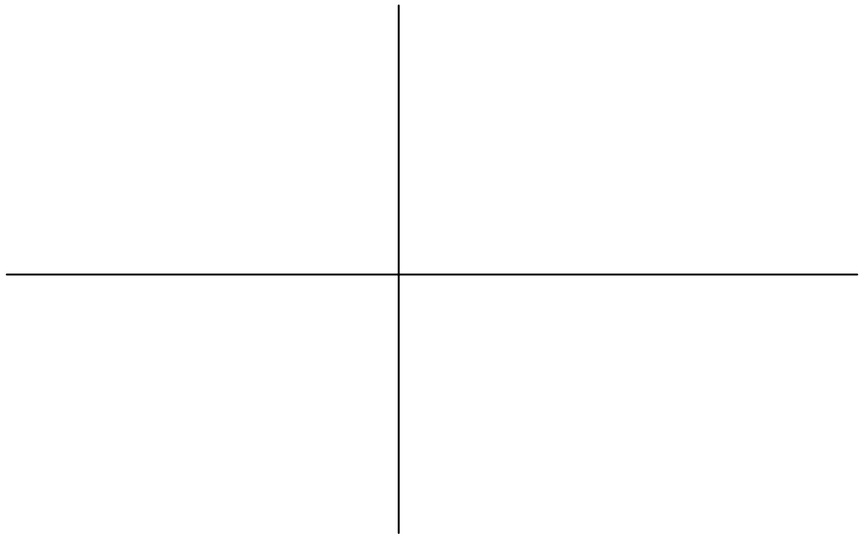
Example 5: Find the solution: $4x^2 - 4x + 5 = 0$

Section 5.2 – Trigonometric Form of Complex Numbers

I. The Complex Plane

The **complex plane** is the coordinate system where we graph ____________________ ________________________.

On the complex plane shown below, (a) label the real axis, (b) label the imaginary axis, and (c) plot and label the complex numbers $-2-3i$, $4+i$, 5, and $2i$.



The absolute value of the complex number $z = a+bi$ is given by $|a+bi|$ = ________________________________.

II. Trigonometric Form of a Complex Number

The **trigonometric form of a complex number** $z = a+bi$ is z = ______________________________, where

$a =$ __________________________, $b =$ __________________________,

$r =$ __________________________, $\tan\theta =$ ______________________.

Example 1: Write $z = 3-3i$ in trigonometric form.

Example 2: Write $z = 5(\cos 120° + i\sin 120°)$ in standard form.

III. Multiplication and Division of Complex Numbers

Let $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$ be complex numbers. Then:

$z_1 z_2 =$ ________________________________

$\frac{z_1}{z_2} =$ __________________________________

Example 3: Multiply: $8(\cos 88° + i\sin 88°) \cdot 12(\cos 112° + i\sin 112°)$

Example 4: Divide: $\dfrac{12cis\frac{2\pi}{3}}{4cis\frac{11\pi}{6}}$

Section 5.3 – **DeMoivre's Theorem**

I. Powers of Complex Numbers

DeMoivre's Theorem

If $z = rcis\theta$ and n is a positive integer, then z^n = ____.

Example 1: Write in standard form: $\left(\cos 240° + i\sin 240°\right)^{12}$

II. Roots of Complex Numbers

The complex number $u = a + bi$ is an **nth root** of the complex number z if ____________________________________.

For a positive integer n, the complex number $z = r\left(\cos\theta + i\sin\theta\right)$ has _______ distinct nth roots given by

$$w_k = \sqrt[n]{r}\left(cis\frac{\theta + 360°k}{n}\right), \qquad k = 0,1,2,...,n-1.$$

When k exceeds n-1, the roots will ____________________.

Example 2: Find the four fourth roots of $1+i$.

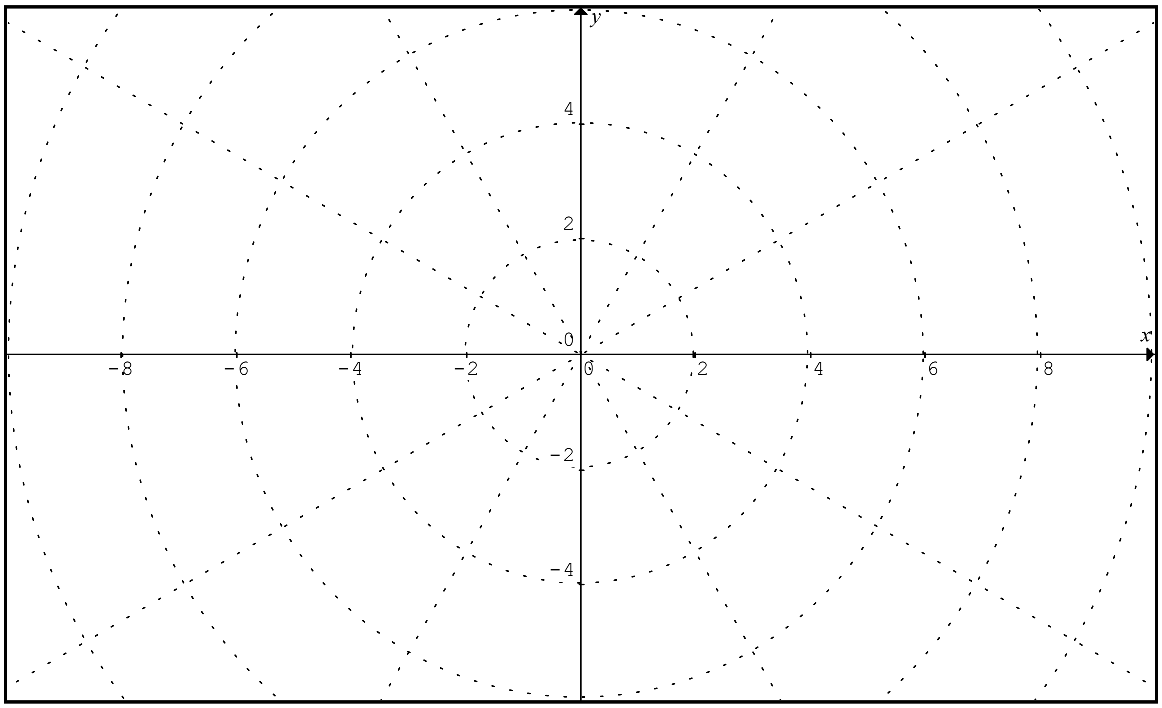
Section 6.5 – Introduction to Polar Coordinates

I. Introduction

A _________________ coordinate system is formed by drawing a horizontal ray. The ray is called the ______________ _______________, and the endpoint of the ray is called the __________________.

A point $P(r,\theta)$ in the plane is located by specifying a distance _______ from the pole and angle θ measured from the polar axis to the line segment OP. The angle can be measured in degrees or radians.

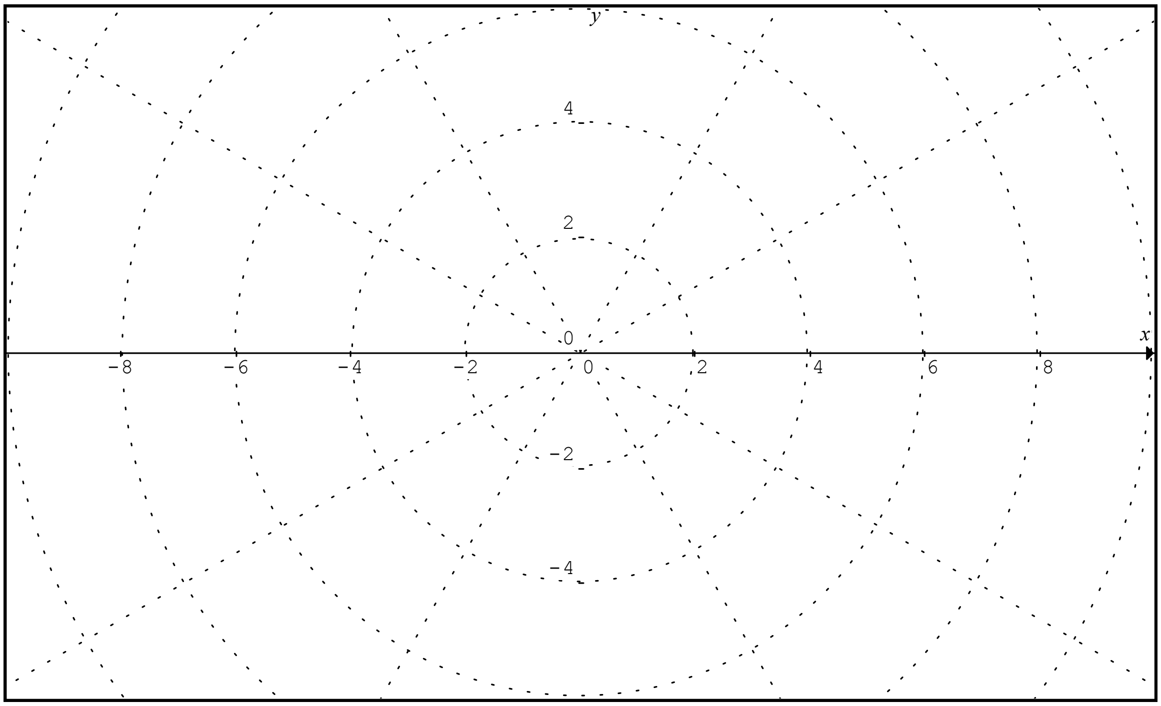
Example 1: Plot the points on the polar coordinate system. a. $(2, 11\pi/4)$ b. $(-1,\pi/3)$ c. $(-4,0)$



In a rectangular coordinate system, there is a one-to-one correspondence between the points in the plane and the ordered pairs (x,y). **This is not true for a polar coordinate system!** Infinitely many ordered pairs correspond to each point in a polar coordinate system.

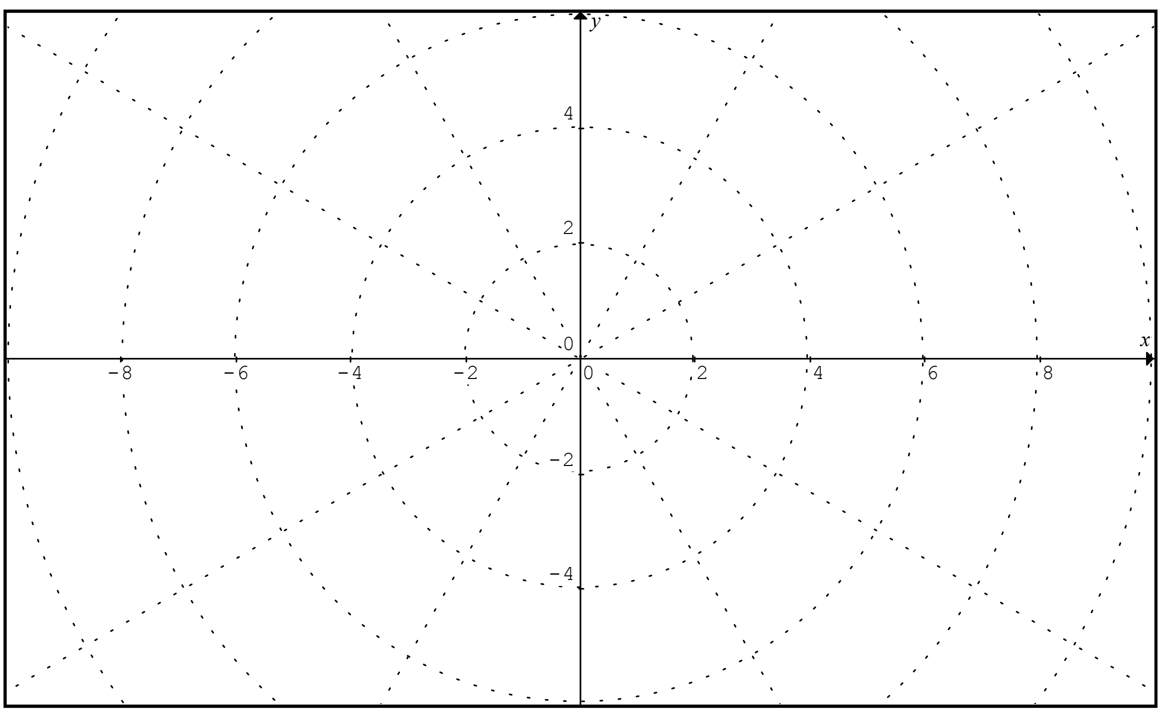
Example 2: Find another polar representation of the point $(4, \pi/6)$.

Example 3: Graph (a) $\theta = \pi/3$ (b) $r\sin\theta = 2$ (c) $r\cos\theta = 4$



Note that any equation that has the same form as one of those given in example 3 will be a line.

Example 4: Graph (a) $r = 2$ (b) $r = 3\sin\theta$ (c) $r = 4\cos\theta$

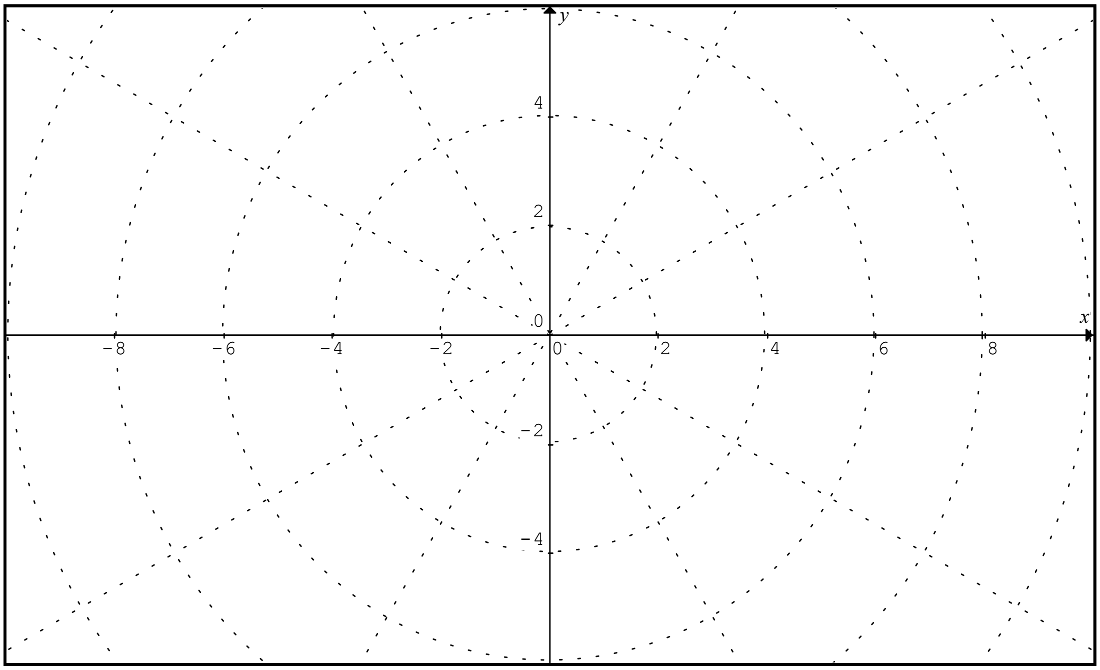


Note that any equation that has the same form as one of those given in example 4 will be a circle. Note that graph (a) is symmetric with respect to the pole, graph (b) is symmetric with respect to $\theta = \pi/2$, and graph (c) is symmetric with respect to $\theta = 0$.

II. Special Polar Graphs

The graph of an equation of the form $r = a \pm a\sin\theta$ or $r = a \pm a\cos\theta$ is a ___________________________. If the equation involves $\sin\theta$ it will be symmetric with respect to the line θ = π/2 and if the equation involves $\cos\theta$ the graph will be symmetric with respect to the line θ = 0.

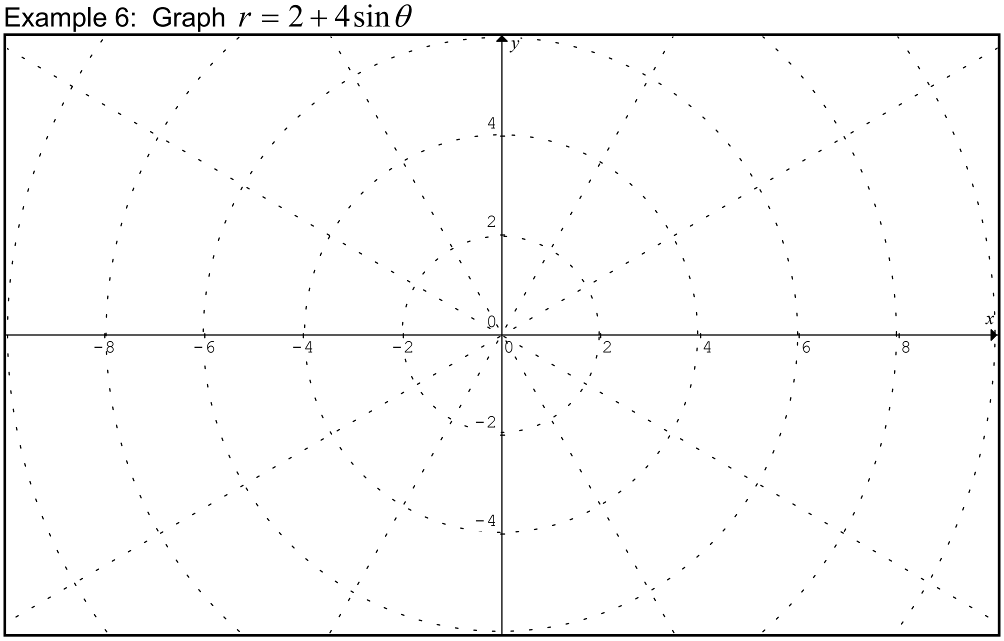
Example 5: Graph $r = 2 + 2\sin\theta$.



The graph of an equation of the form $r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$ is a _________________________.

If $a \geq 2b$ then it will be a convex limacon (no dimple).
If $b < a < 2b$ then it will have a dimple.
If $a < b$ then it will have an inner loop.
If $a = b$ then it is a cardiod.

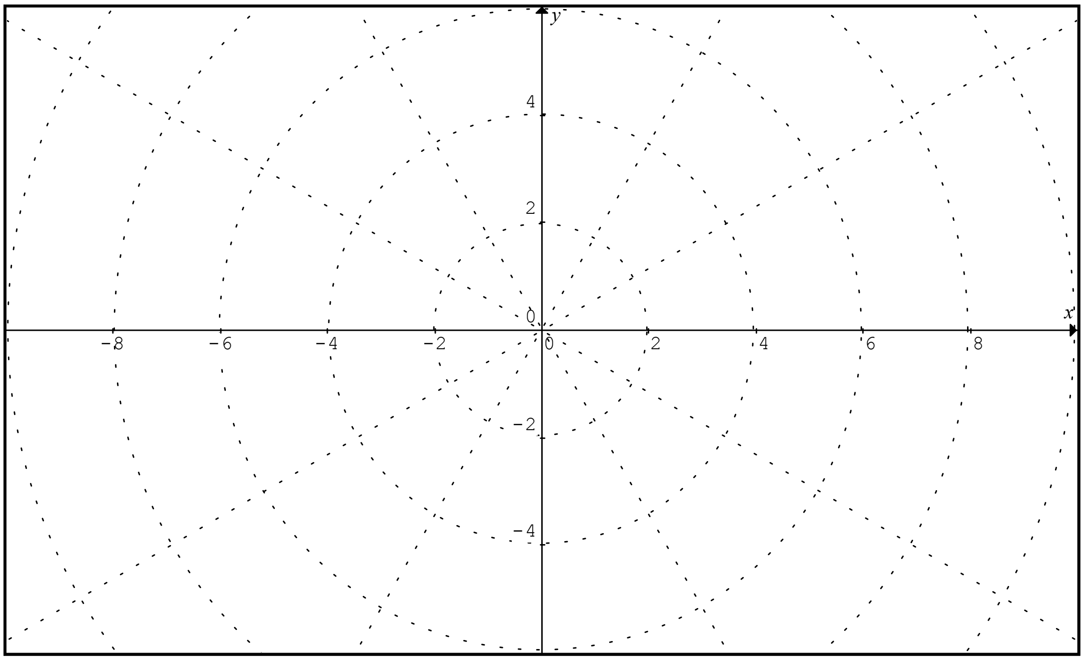
Example 6: Graph $r = 2 + 4\sin\theta$



The graph of an equation of the form $r = a\cos n\theta$ or $r = a\sin n\theta$ are ____________________ curves.

When n is an even number , the number of petals is 2n. When n is an odd number, the number of petals is n.

Example 7: Sketch the graph of $r = 4\sin 5\theta$.



III. Coordinate Conversion

Polar coordinates (r,θ) are related to the rectangular coordinates (x, y) as follows…

Example 8: Convert the polar coordinates (3,3π/2) to rectangular coordinates.

To convert a rectangular equation to polar form,…

Example 9: Find the rectangular equation corresponding to the polar equation $r = \frac{-5}{\sin\theta}$.