Relative Rate of Change of $f(t) = \frac{d}{dt} \ln f(t) = \frac{f'(t)}{f(t)}$

If application, if $f(t)$ is the price of an item at time $t$, then the rate of change is $f'(t)$, and the relative rate of change is $\frac{f'(t)}{f(t)}$, the derivative divided by the function. We sometimes refer to $f'(t)$ as the “absolute” rate of change to distinguish between the two. Relative rates of change give us a means of comparing absolute rates of change for incomparable items. Furthermore, since it is a ratio or percent it will not depend on the units of the function whether its dollars, francs, pounds, etc.

Example:

#4

#12

**Elasticity of Demand**

Typically people want to maximize revenue which depends on price and quantity sold. In general, if the price of an item goes up, the demand and resulting sales go down. On the other hand, if there is an overabundance of an item and price goes down, quantity sold or demand goes up.

The idea of elasticity of demand was devised to answer the question of whether an increase/decrease in price for a particular item and the resulting decrease/increase in demand will compensate for each other. On an intuitive level elasticity of demand measures how responsive demand is to a price change. If demand is very responsive we consider it elastic. If it is not very responsive, in other words, even if the price goes up or down appreciably demand stays about the same, we consider it inelastic.

In general there is an inverse relationship between price and demand for an item. Price goes up demand goes down and vice versa. A demand function gives the relationship between the price $p$ of an item and the quantity $x$ that will be sold at that price.

Demand Function—
The demand function $x = D(p)$ gives the quantity $x$ of an item that will be demanded by consumers at price $p$. 
Elasticity of Demand

\[
E(p) = \frac{\text{Relative rate of change of demand}}{\text{Relative rate of change of price}} = \frac{-pD'(p)}{D(p)}
\]

Demand is elastic if \( E(p) > 1 \) and inelastic if \( E(p) < 1 \). Since elasticity depends on relative rates of change it does not depend on the units of the demand function and can, therefore, be compared between different products.

To increase Revenue:
- Raise prices if demand is inelastic (\( E < 1 \))
- Lower prices if demand is elastic (\( E > 1 \))

For revenue to be a maximum, elasticity of demand = 1 which is called unitary elastic.

Example:
- #16
- #20
- #28