Departmental Final Exam Review for Trigonometry  
Part 1: Chapters 1, 2, and 3

Departmental Objectives:
1. Analyzing and classifying angles, converting between units, using the Arc Length Formula, and using the relationship between linear and angular speed. [Sections: 1.1, 3.1, 3.2, 3.4]
2. Defining, applying, and finding the exact values of the six trigonometric functions.  
   [Sections: 1.3, 1.4, 2.1, 2.2, 2.3, 2.4, 2.5, 3.3]

Note: Corresponding MML numbers listed at the end of the problem in [ ]

Chapter 1: Trigonometric Functions

1. Find (a) the complement and (b) the supplement of $61^\circ$. Show all work and / or support your answer. [1.1.9]

2. Find the smallest positive coterminal angle for $-1234^\circ$. Show all work and / or support your answer. [1.1.91]

3. Sketch the angle $\theta = 553^\circ$ in standard position. What quadrant does it terminate in? Draw an arrow representing the correct amount of rotation. Find the measure of two other angles, one positive and one negative, that are coterminal with $\theta$. Show all work and / or support your answer. [1.1.111]

4. A tire is rotating 320 times per minute. Through how many degrees does it rotate in 8 seconds? Show all work and / or support your answer. [1.1.133]

5. Find the exact values of the six trigonometric functions for the angle in standard position having the point $(1,3)$ on its terminal side. Rationalize denominators if applicable. Show all of your work. [1.3.15]

6. Suppose that the point $(x, y)$ is in quadrant IV. Decide whether the ratio $\frac{y}{r}$ is positive or negative. Show all work and / or support your answer. [1.3.27]

7. Use the appropriate reciprocal identity to find $\tan \theta$, if $\cot \theta = \frac{-\sqrt{3}}{3}$. Rationalize denominators when applicable. Show all work and / or support your answer. [1.4.9]

8. Identify the quadrant or quadrants for the angle satisfying the condition $\cos \theta < 0$, $\csc \theta > 0$. Show all of your work. [1.4.35]

9. Give the signs of the sine, cosine, and tangent functions for $472^\circ$. Show all work and / or support your answer. [1.4.27]

10. Use identities to find $\sin \theta$, if $\sec \theta = 2$, with $\theta$ in quadrant IV. Show all work and / or support your answer. [1.4.63]

11. Find the exact values of the remaining trigonometric functions of $\theta$ if $\sin \theta = \frac{\sqrt{3}}{7}$ in Quadrant III. Show all work and / or support your answer. [1.4.71]
Chapter 2: Acute Angles and Right Triangles

12. For each trigonometric function in the first row choose its value in the second row. Do not use a calculator. Show all work and / or support your answer. [2.1.5, 7, 9]
   a. \( \csc 60^\circ \)  
   b. \( \cos 45^\circ \)  
   c. \( \tan 30^\circ \)  
   A. \( \frac{\sqrt{3}}{3} \)  
   B. \( \frac{\sqrt{2}}{2} \)  
   C. \( \sqrt{2} \)  
   D. \( \sqrt{3} \)  
   E. \( \frac{2\sqrt{3}}{3} \)  
   F. \( \frac{1}{2} \)

13. ABC is a right triangle with side lengths \( a = 3 \) and \( b = 5 \) and a \( 90^\circ \) angle at C. Find the length of side \( c \) and the six trigonometric function values for angle B. Do not use a calculator. Show all work and / or support your answer. [2.1.11]

14. Write \( \csc 25.4^\circ \) in terms of its cofunction. Show all work and / or support your answer. [2.1.21]

15. Give the exact value of (a) sec \( 45^\circ \) and (b) cot \( 30^\circ \). Show all work and / or support your answer. [2.1.53, 59]

16. In the diagram below, find the exact value of each part labeled with a variable. Show all work and / or support your answer. [2.1.81]

17. Match each angle in the first row with its reference angle in the second row. Show all work and / or support your answer. [2.2.1, 3]
   a. \( 284^\circ \)  
   b. \( -225^\circ \)  
   c. \( 780^\circ \)  
   A. \( 30^\circ \)  
   B. \( 14^\circ \)  
   C. \( 45^\circ \)  
   D. \( 76^\circ \)  
   E. \( 60^\circ \)  
   F. \( -45^\circ \)

18. Find exact values of the six trigonometric functions for: (a) \( 480^\circ \) (b) \( -135^\circ \) (c) \( 330^\circ \). Show all work and / or support your answer. [2.2 23, 27, 31]

19. Find all values of \( \theta \), if \( \theta \) is in the interval \([0^\circ, 360^\circ]\) and has the given function value. Show all work and / or support your answer. [2.2.85, 89]
   a. \( \sec \theta = -\sqrt{2} \)  
   b. \( \tan \theta = \sqrt{3} \)

20. Use a calculator to find a decimal approximation for sec \( 147^\circ 11' \). Round your answer to four decimal places. Show all work and / or support your answer. [2.3.7]

21. Use a calculator to find a value of \( \theta \) in the interval \([0^\circ, 90^\circ]\) for which \( \cot \theta = 1.2575516 \). Show all work and / or support your answer. [2.3.27]

22. Solve the right triangle shown below. Show all work and / or support your answer. Answers should be shown with the correct number of significant digits. [2.4.11]
23. Solve the right triangle ABC where $a = 76.4$ yd, $b = 39.3$ yd, and $C = 90^\circ$. Give angles in degrees and minutes. Show all work and / or support your answer. [2.4.29]

24. Suppose that the angle of elevation of the sun is $23.4^\circ$. Find the length of the shadow cast by Dot Peterson, who is 5.75 feet tall. [2.4.51]°

25. A company safety committee has recommended that a floodlight be mounted in a parking lot so as to illuminate the employee exit. Find the angle of depression of the light to the nearest minute. [2.4.53]

26. Two ships leave port at the same time. The first ship sails on a bearing of $40^\circ$ at 18 knots (nautical miles per hour) and the second at a bearing of $130^\circ$ at 26 knots. How far apart are they after 1.5 hrs? Show all work and / or support your answer. [2.5.17]

27. The bearing from Elliston to Pangle is N 42° E. The bearing from Pangle to Themster is S 48° E. A car driven at 60 mph takes 1 hr to go from Elliston to Pangle and 1.8 hr to go from Pangle to Themster. Find the distance from Elliston to Themster. Show all work and / or support your answer. [2.5.25]

28. To measure a stone face carved on the side of a mountain, two sightings 750 feet from the base of the mountain are taken. If the angle of elevation to the bottom of the face is $34^\circ$ degrees and the angle of elevation to the top is $36^\circ$ degrees, what is the height of the stone face? [2.5.35]

Chapter 3: Radian Measure and Circular Functions

29. Convert (a) $345^\circ$, and (b) $240^\circ$ 45' to radians. Show all work and / or support your answer. On part a, express your answer in terms of $\pi$. On part b, round your answer to three decimal places. [3.1.11, 49]

30. Convert (a) $-\frac{4\pi}{3}$, (b) $\frac{7\pi}{4}$, and to degrees. Show all work and / or support your answer. [3.1.31, 39]

31. Find the exact value of $\cos \left(-\frac{5\pi}{6}\right)$. Do not use a calculator. Show all work and / or support your answer. [3.1.85]

32. Find the length to three significant digits of each arc intercepted by a central angle $\theta$ in a circle of radius $r$. [3.2.13, 17]
   a. $r = 1.38$ ft, $\theta = \frac{5\pi}{6}$ radians
   b. $r = 15.1$ in., $\theta = 210^\circ$

33. Find the area of a sector of a circle having radius $r = 90.0$ km and central angle $\theta = 270^\circ$. Show all work and / or support your answer. [3.2.57]

34. Find the exact values of (a) $\sin \theta$, (b) $\cos \theta$, and (c) $\tan \theta$ for $\theta = \frac{-3\pi}{2}$. Do not use a calculator. Show all work and / or support your answer. [3.3.5]
35. Find the exact circular function value for (a) \( \tan \frac{17\pi}{4} \), (b) \( \csc \frac{11\pi}{6} \). Show all work and / or support your answer. [3.3.11, 21]

36. Find the value of \( t \) in the given interval that has the given circular function value. Do not use a calculator. Show all work and / or support your answer. [3.3.61, 63, 67]
   a. \( \left[ \frac{\pi}{2}, \pi \right] ; \cos t = -\frac{1}{2} \)
   b. \( \left[ \frac{3\pi}{2}, \pi \right] ; \tan t = \frac{\sqrt{3}}{3} \)
   c. \( \left[ \frac{3\pi}{2}, 2\pi \right] ; \sin t = -\frac{\sqrt{2}}{2} \)

37. Suppose that point P is on a circle with radius 10 cm, and ray OP is rotating with angular speed \( \frac{\pi}{18} \) radians per second. [3.3.3]
   a. Find the angle generated by P in 6 sec. Show all work and / or support your answer.
   b. Find the distance traveled by P along the circle in 6 sec. Show all work and / or support your answer.
   c. Find the linear speed of P. Show all work and / or support your answer.

38. Given \( \omega = \frac{2\pi}{3} \) radians per sec and \( t = 3 \) sec, use the formula for angular speed to find \( \theta \). Show all work and / or support your answer. [3.4.7]

39. Given \( v = 9 \) m per sec and \( r = 5 \) m, use the formula for linear speed to find \( \omega \). Show all work and / or support your answer. [3.4.17]

40. Given \( s = \frac{216\pi}{5} \) yd, \( r = 9 \) yd, and \( \omega = \frac{2\pi}{5} \) radians per second, use the formulas for arc length and angular speed to find \( t \). Show all work and / or support your answer. [3.4.23]

41. The tires of a bicycle have radius 13 in. and are turning at the rate of 200 rpm. How fast is the bicycle traveling in mph? Show all work and / or support your answer. [Hint: \( 5280 \text{ ft} = 1 \text{ mile.} \) [3.4.37]
Answers

Chapter 1: Trigonometric Functions

1. $29^\circ; 119^\circ$
2. $206^\circ$
3. III; $193^\circ; -167^\circ$
4. $15.360^\circ$
5. \[ \begin{align*} 
\sin \theta &= \frac{-1}{2} \\
\cos \theta &= \frac{-\sqrt{3}}{2} \\
\tan \theta &= \frac{-\sqrt{3}}{3} \\
\csc \theta &= -2 \\
\sec \theta &= \frac{2\sqrt{3}}{3} \\
\cot \theta &= \frac{-\sqrt{3}}{3} 
\end{align*} \]
6. $\frac{y}{r}$ will be negative because $y$ is negative in Q IV and $r$ is always positive.
7. $-\frac{3\sqrt{5}}{5}$
8. II
9. $\sin \theta$ is positive; $\cos \theta$ and $\tan \theta$ are negative.
10. $-\frac{\sqrt{3}}{2}$
11. $\cos \theta = -\frac{\sqrt{7}}{3}$, $\tan \theta = \frac{138}{46}$, $\csc \theta = -\frac{7\sqrt{46}}{3}$, $\sec \theta = \frac{\sqrt{138}}{46}$, $\cot \theta = \frac{3}{\sqrt{3}}$
12. $E; B; A$
13. $c = \sqrt{34}$
14. sec 64.6
15. $\sqrt{2}; \sqrt{3}$
16. $a = 12; b = d = 12\sqrt{3}; c = 12\sqrt{6}$
17. D; C; E
18a. $\sin 480^\circ = \frac{\sqrt{3}}{2}$, $\cos 480^\circ = -\frac{1}{2}$, $\tan 480^\circ = -\sqrt{3}$
19. $135^\circ; 225^\circ; 60^\circ; 240^\circ$
20. -1.1899
21. $38.491580^\circ$
22. $38.8^\circ; 154^\circ; 198^\circ$
23. $85.9; 62^\circ 50^\prime; 27^\circ 10^\prime$
24. $13.3$ ft.
25. $37^\circ 35^\prime$
26. $47$ nautical miles
27. $120$ miles
28. $39.03$ ft.
29. $\frac{23\pi}{12}; 4.202$
30. $-240^\circ; 315^\circ$

Chapter 2: Acute Angles and Right Triangles

Chapter 3: Radian Measure and Circular Functions
31. \(-\frac{\sqrt{3}}{2}\)

32. 3.61 ft; 55.3 in

33. 19085.2 km²

34. 1; 0; undefined

35. 1; -2

36. \(\frac{2\pi}{3}; \frac{7\pi}{6}; \frac{7\pi}{4}\)

37. \(\frac{\pi}{3}; 10\pi/3\) cm; \(5\pi/9\) cm per sec

38. \(2\pi\)

39. \(\frac{9}{5}\) rad per sec

40. 12 sec

41. 15.5 mph