1.4 Equations of Lines and Modeling

I. Summary of Key Concepts about Lines

What makes a line different from a curve? A line is straight. It has a constant slope.

A. Slope

The slope (m) of the line containing the points (x_1, y_1) and (x_2, y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$

B. Vertical Lines

Vertical lines cut the x-axis. Their equations are in the form x = a.

The slope of a vertical line is undefined.

Vertical lines are <u>not</u> functions.

C. Horizontal Lines

Horizontal lines cut the y-axis. Their equations are in the form y = b.

The slope of a horizontal line is zero.

Horizontal lines are Constant (zero degree) Functions.

Example 1:
Write the equations of the horizontal and vertical lines that pass through the point $\left(\frac{2}{11}, -1\right)$. (#51 p. 116)

Vertical line: $\boxed{x = \frac{2}{11}}$
(cuts x-axis)

Horizontal line: $\boxed{y = -1}$
(cuts y-axis)

D. Oblique Lines

Oblique lines cut the x-axis and the y-axis. They are true Linear (1st degree) Functions.

1. The **Slope-Intercept Form** (SIF) of the Equation of a Line is y = mx + b.
   m is the slope of the line and (0, b) is the y-intercept of the line.

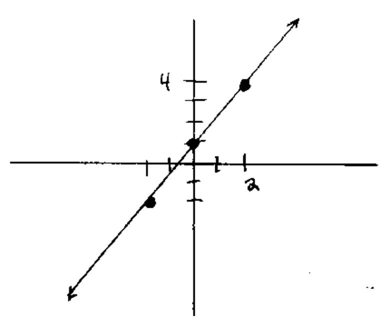
   To graph the equation of a line given in slope-intercept form, we plot the y-intercept on the y-axis. Then, if the slope is positive, we go up Δy and right Δx [or down Δy and left Δx] to find a second point and again to find a third point. Or, if the slope is negative, we go down Δy and right Δx [or up Δy and left Δx] to find a second point and again to find a third point.

Example 2: Graph the equation $y = \frac{3}{2}x + 1$ using the slope and the y-intercept. (#38 p. 115)

$m = \frac{3}{2}$ 3 up, 2 right   y-intercept: (0, 1)

or $m = \frac{-3}{-2}$ 3 down, 2 left

	x	y	
2 rt	2	4	3 up
	0	1	
2 left	-2	-2	3 down



2. The **Standard Form** of the Equation of a Line is Ax + By = C.

   To graph the equation of line given in standard form, we find the x-intercept by plugging 0 in for y and solving for x; we find the y-intercept by plugging zero in for x and solving for y; and we find a third point by plugging in either an x or a y of our choice and then solving for the other variable.

   To find the slope and y-intercept of the standard form of a line, we can put it in slope-intercept form by isolating y.

Example 3: Graph the equation 2x + 3y = 15. Find the slope and the y-intercept. (#42 p. 115)

	x	y
x-int	$\frac{15}{2}$	0
y-int	0	5
3rd pt	3	3

↑ my choice

$2(3) + 3y = 15$

$6 + 3y = 15$

$3y = 9$

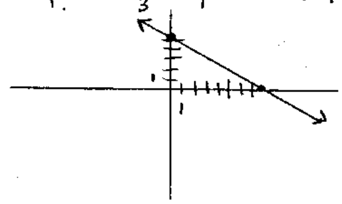
$y = 3$

$2x + 3y = 15$

$\frac{3y}{3} = \frac{-2x}{3} + \frac{15}{3}$

$y = -\frac{2}{3}x + 5$

slope: $-\frac{2}{3}$ y-int: (0, 5)



3. The **Point-Slope Form** (PSF) of the Equation of a Line is $y - y_1 = m(x - x_1)$ where (x_1, y_1) is a point on the line and m is the slope of the line.

E. Parallel and Perpendicular Lines

Parallel lines have the same slope. If the slope of line A is $m = \frac{a}{b}$, the slope of a line parallel to line A is $m_{\parallel} = m = \frac{a}{b}$.

Perpendicular lines have negative reciprocal slopes. If the slope of line A is $m = \frac{a}{b}$, the slope of a line perpendicular to line A is $m_{\perp} = -\frac{1}{m} = -\frac{b}{a}$.

II. How to Write the Equation of a Line

To write the equation of a line, we must know three things: 1) a point (x_1, y_1), 2) the slope (m), and 3) a formula for the equation of a line.

A. How to write the equation of a line if you are given a point and the slope:

<u>Method 1: Using the Point-Slope Form</u>

Plug x_1, y_1, & m into the point-slope form. Simplify. Isolate y.

<u>Method 2: Using the Slope-Intercept Form</u>

Plug x_1, y_1, & m into the slope-intercept form to find b.
Then, plug m & b into the slope-intercept form to write the equation.

Don't forget to write the EQN!

} You can use either method. You do not have to do both.

Example 4: Write the slope-intercept equation for a line which has slope $m = -\frac{3}{8}$ and which passes through the point (5, 6). (#26 p. 115)

Method 1

$x_1 = 5 \quad y_1 = 6 \quad m = \frac{-3}{8}$

$y - y_1 = m(x - x_1)$

$y - 6 = -\frac{3}{8}(x - 5)$

$y - 6 = -\frac{3}{8}x + \frac{15}{8}$

$\boxed{y = -\frac{3}{8}x + \frac{63}{8}}$

Workspace:

$\frac{15}{8} + 6 =$

$\frac{15}{8} + \frac{48}{8} =$

$\frac{63}{8}$

To check EQN, plug in x of pt.

$y = -\frac{3}{8}(5) + \frac{63}{8} = \frac{48}{8} = 6$ ✓

Method 2

$x = 5 \quad y = 6 \quad m = \frac{-3}{8}$

$y = mx + b$

$6 = -\frac{3}{8}(5) + b$

$6 = -\frac{15}{8} + b$

$\frac{63}{8} = b$

$\boxed{y = -\frac{3}{8}x + \frac{63}{8}}$

Don't forget to write EQN!

B. How to write the equation of a line if you are given two points

Find the slope ($m = \frac{y_2 - y_1}{x_2 - x_1}$) first. Then use one of the methods from A to write the equation.

Example 5:

Write the slope-intercept equation for a line which passes through the points (2, -1) and (7, -11). (#32 p. 115)

$x_1 \ y_1 \quad x_2 \ y_2$

$m = \frac{-11-(-1)}{7-2} = \frac{-10}{5} = -2$

Method 1

$x = 2 \quad y = -1 \quad m = -2$

$y - (-1) = -2(x - 2)$

$y + 1 = -2x + 4$

$\boxed{y = -2x + 3}$

Method 2

$x = 2 \quad y = -1 \quad m = -2$

$-1 = -2(2) + b$

$-1 = -4 + b$

$3 = b$

$\boxed{y = -2x + 3}$

Don't forget to write EQN!

C. How to write the equation of a line if you are given a point and the equation of a parallel line

Solve the given equation for y to find its slope (m). Then, using that slope and the given point, use one of the methods from A to write the equation.

Example 6:

Write the slope intercept equation for the line which passes through the point (-4, -5) that is parallel to the line 2x + y = -4. (#64 p. 116)

$2x + y = -4 \rightarrow y = -2x - 4 \quad m = -2 \quad m_{\parallel} = -2$

↑ m of parallel line

Method 1

$x_1 = -4 \quad y_1 = -5 \quad m = -2$

$y - (-5) = -2(x - (-4))$

$y + 5 = -2(x + 4)$

$y + 5 = -2x - 8$

$\boxed{y = -2x - 13}$

Method 2

$x = -4 \quad y = -5 \quad m = -2$

$-5 = -2(-4) + b$

$-5 = 8 + b$

$-13 = b$

$\boxed{y = -2x - 13}$

Don't forget to write EQN.

D. How to write the equation of a line if you are given a point & the equation of a perpendicular line:

Solve the given equation for y to find its slope (m). Then, using $m_\perp = -\frac{1}{m}$ and the given point, use one of the methods from A to write the equation.

Example 7:

Write the slope intercept equation for the line which passes through the point (-4, -5) that is perpendicular to the line 2x + y = -4. (#64 p. 116) $2x + y = -4 \rightarrow y = -2x - 4$

$m = -2 \quad m_\perp = \frac{1}{2}$

Method 1

$x = -4 \quad y = -5 \quad m = \frac{1}{2}$

$y - (-5) = \frac{1}{2}(x - (-4))$

$y + 5 = \frac{1}{2}(x + 4)$

$y + 5 = \frac{1}{2}x + 2$

$\boxed{y = \frac{1}{2}x - 3}$

Method 2

$x = -4 \quad y = -5 \quad m = \frac{1}{2}$

$-5 = \frac{1}{2}(-4) + b$

$-5 = -2 + b$

$-3 = b$

$\boxed{y = \frac{1}{2}x - 3}$

Don't forget to write EAN!

III. Regression Analysis on a TI-82, 83, or 84 Graphing Calculator

1. Clear Old Data [Omit Step 1 if there is no old data in L1 and/or L2.]

   STAT 1:Edit ▲ (highlight L1) Clear Enter ► ▲ (highlight L2) Clear Enter ◄

2. Enter Data

   STAT 1:Edit (Enter input (x) values in L1 and output (y) values in L2.) Make sure every entry is correct.

Example 8:

Average annual tuition and fees for in-state students at public 4-year colleges for selected years are shown in the table. Using x = 0 for 1990, enter the x-values (years since 1990) in L1 and the y-values (tuition and fees) in L2.

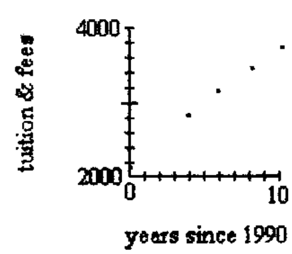
Year	**Years since 1990 (x) L1**	**Tuition & Fees (y) L2**
1990	0	2035
1994	4	2820
1996	6	3151
1998	8	3486
2000	10	3774

3. Create a Scatter Plot of the Data

   y = CLEAR (Clear out or turn off any equations.)

   2nd y = 1:Plot 1 (to select Plot 1) Enter (to turn Plot 1 on)

   ZOOM 9:ZoomStat



4. <u>Run Regression</u>

STAT ► CALC

select appropriate regression LinReg (ax + b) **QuadReg CubicReg QuartReg etc.**

Enter (to execute regression analysis)

LinReg

$y = ax + b$	Record the model as an equation (you may round to three decimal places).
$a = 174.3986486$	**$y = 174.399x + 2076.567$**
$b = 2076.567568$	If given, record the correlation coefficient, r (do not round).
$r^2 = .9961354296$	**$r = .9980658443$** (We will not be using r^2, the correlation of determination.)
$r = .9980658443$	Tell the goodness of fit (The closer r is to 1 or -1, the better the fit). **Very good fit**

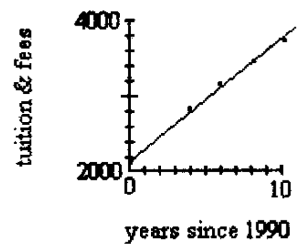
TI-83 and TI-84 users

If you do not see r and r^2, do the following: Press 2nd 0 (catalog) and scroll down until you see DiagnosticOn. Hit enter to select that command and then hit enter again to execute it. Run the regression again. You should now see r and r^2.

5. <u>Graph the Model with the Data</u>

y = VARS 5:Statistics ►► RegEQ GRAPH

Create an accurate sketch of the model with the data.



If the type of regression you ran does not give a correlation coefficient, you can assess the goodness of fit by examining the graph of the model with the data. The more data points the model goes through or touches, the better the fit.

6. <u>To Find the Output for a Specified Input</u>

2nd Graph (Table) Enter appropriate input.
For example, by plugging in 18 we can get the projected cost of tuition and fees in 2008.
y(18) = $5215.70

7. <u>Turn the Plotter Off</u>

2nd y = 4 Enter (It should say PlotsOff Done.)