Math in the Real World:
College Algebra Extra Credit Problems

For each problem, a correct solution with appropriate support work is worth one point.

1. The growth of a human fetus more than 12 weeks old can be approximated by the formula
   \[ L = 1.53t - 6.7, \]
   where \( L \) is the length (in centimeters) and \( t \) is the age (in weeks). Prenatal length can be
determined by ultrasound. Approximate the age of a fetus whose total length is 28 centimeters. Show
all of your work or explain how you came up with your solution.

2. In 1990, around 115 billion dollars was invested globally in telecommunications infrastructure. In 1994,
   around 145 billion was spent. Assuming that global investment in telecom infrastructure is growing at a
constant rate, write a linear model which gives investment \( I \) in billions of dollars as a function of \( t \), the
number of years since 1990. Use your model to predict global telecom investment in 2003.

3. The following table gives the enrollment at NWACC for the Fall semesters from 1990 to 2000.
   Create a scatter plot of the data. Assuming that enrollment is growing at a constant rate, use the
regression capabilities of your calculator to run a linear regression analysis on the data. Let \( x = 0 \)
represent 1990. What is the linear equation which best fits the data (round to three decimal places)?
What is the correlation coefficient? According to this model, what will enrollment be in the Fall of
2005? How confident are you about this projection?

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</thead>
<tbody>
<tr>
<td>Enrollment</td>
<td>1232</td>
<td>1632</td>
<td>1889</td>
<td>1972</td>
<td>2037</td>
<td>2244</td>
<td>2941</td>
<td>3240</td>
<td>3542</td>
<td>3923</td>
<td>4095</td>
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4. It is a surprising biological fact that most crickets chirp at a rate that increases as the temperature
increases. For the snowy tree cricket (\textit{Oecanthus fultoni}), the relationship between temperature and
chirp rate is so reliable that this type of cricket is called the thermometer cricket. One can estimate the
temperature (in °F) by counting the number of times a snowy tree cricket chirps in 15 seconds and
adding 40. For instance, if we count 20 chirps in 15 seconds, then a good estimate of the temperature
would be \( 20 + 40 = 60^\circ \text{F} \). The rule used to find the estimated temperature \( T \) (in °F) based on the
measured chirp rate \( R \) (in chirps per minute) is an example of a function. Describe this function in four
different ways: using words, a table, a graph, and a formula.

5. The difference quotient \( \frac{s(a + \Delta t) - s(a)}{\Delta t} \) can be used to evaluate the average velocity of an object rolling
down a ramp. \( s \) is the distance the object travels, \( t \) is the time in seconds after the object is released, \( a \) is
the start time, and \( \Delta t \) is the length of the time interval (finish time - start time). If the distance traveled
by a ball rolling down a ramp is given by the function \( s(t) = 4t^2 \), what is the average velocity of the ball
over the time interval from \( t = 3 \) to \( t = 3.01 \)? Show all of your work or explain how you came up with
your solution.

6. The concentration \( C(t) \), in milligrams per liter, of a medication in a patient’s blood is given by the
function \( C(t) = 25t^3 - 150t^2 + 225t \) where \( t \) is time in hours after the medication was taken. Use the
difference quotient \( \frac{C(a + \Delta t) - C(a)}{\Delta t} \) to find the average rate of change of the concentration over the
time interval from \( t = 1 \) to \( t = 1.25 \). After how many hours will the drug be totally eliminated from the
bloodstream? Show all of your work or explain how you came up with your solutions.

7. Captain Janeway left the holodeck at 7:45 to meet Tuvok, her Chief of Security, on the Bridge. After
walking for 3 minutes, she realized she had forgotten her tricorder and returned to get it. She picked up
the tricorder and resumed her walk, arriving on the Bridge at 8:00. After 15 minutes the discussion was
over, and Janeway returned to the holodeck. Graph Janeway’s distance from the holodeck as a function
of time. Determine on which time intervals the graph is increasing, decreasing, and constant.

8. The velocity of the air that is expelled during a cough can be modeled by \( v = .6r^2 - r^3 \), where \( v \) is
measured in centimeters per second and \( r \) is the radius of the trachea in centimeters. Find the radius of
the trachea, to the nearest .1 centimeter, that maximizes the velocity of the air. Physical considerations of the structure of the trachea indicate that \( .3 \leq r \leq .6 \) is an appropriate domain for \( r \). Show all of your work or explain how you came up with your solution.

9. A coed soccer team has 16 players. Write a function \( g \) that describes the number of girls on the team if \( x \) of the players are boys. If \( f(x) = x \), what transformations of function \( f \) would be necessary to obtain function \( g \)? Show all of your work or explain how you came up with your solution.

10. A fire has started in a dry open field and is spreading in the form of a circle. If the radius of the circle increases at the rate of 6 ft/min, express the total fire area \( A \) as a function of time \( t \) (in minutes). Show all of your work or explain how you came up with your solution.

11. In harvesting 10% of a rainforest, logging companies will often destroy 50% of the unharvested area just to get to the valuable trees. If the function \( u = .9x \) gives the unharvested area with the valuable timber removed, and function \( w = .5u \) gives the amount of unharvested area that is destroyed and wasted, write a function which gives the wasted area \( w \) as a function of the original area \( x \).

12. The formula \( P = 20 + .4t \) gives the population (\( P \), in thousands) of a certain town as a function of time \( t \) (number of years since 1970 since \( t = 0 \) corresponds to 1970). Find the inverse function. What does it tell us?

13. In 1974, at the tomb of an emperor near Xian, Chinese workers found three pits that contained life-size sculptures of warriors. The largest rectangular pit has a length that is 40 yards longer than three times its width. If the perimeter is 640 yards, what are the length and width of the pit? Show all of your work or explain how you came up with your solution.

14. A group of hikers from Tulsa hiked down into the Grand Canyon in 3 hours 30 minutes. Coming back up on a trail that was 4 miles shorter, they hiked 2 mph slower and it took them 1 hour longer. What was their rate going down? Show all of your work or explain how you came up with your solution.

15. The cost of installing insulation in a particular two-bedroom house is $1080. Present monthly heating bills average $60, but the insulation is expected to reduce the heating cost by 10%. How many months will it take to recover the cost of the insulation? Show all of your work or explain how you came up with your solution.

16. The equation \( \frac{1100}{d} + \frac{d}{1100} \) gives the total time (\( T \)) that elapses from the moment an object is dropped into a well to the moment when the sound of its landing is heard. \( d \) is the depth of the well in feet. Use a graphing calculator to determine the depth when \( T = 4 \) seconds. Show all of your work or explain how you came up with your solution.

17. How is dividing by a complex number similar to the process of rationalizing a denominator?

18. Michael Chang, a professional tennis player, skillfully uses the lob as both a defensive and an offensive shot. Chang knows that a well-placed lob can buy the time needed to prepare for the next shot. How long does it take for a tennis ball to hit the earth if it is hit straight upward with a velocity of 60 feet per second from a height of 5 feet? Show all of your work or explain how you came up with your solution.

19. The speed of sound in air at 0°C (or 273 K) is 1087 ft/sec, but this speed increases as the temperature rises. The speed \( v \) of sound at temperature \( T \) in Kelvin is given by \( v = 1087\sqrt{\frac{T}{273}} \). At what temperatures does the speed of sound exceed 1100 ft/sec? Show all of your work or explain how you came up with your solution.

20. The Guinness Book of World Records reports that German shepherds can make vertical leaps over 10 feet when scaling walls. If the distance \( d \) (in feet) off the ground after \( t \) seconds is given by the equation \( d = -16t^2 + 24t + 1 \), for how many seconds is the dog more than 9 feet off the ground? Show all of your work or explain how you came up with your solution.
21. A machinist is producing a circular cylinder on a lathe. The circumference of the cylinder must be 28 inches, with a tolerance of .15 inch. What maximum and minimum radii (to the nearest .001 inch) must the machinist stay between to produce an acceptable cylinder? Show all of your work or explain how you came up with your solution.

22. The national catch of Atlantic Cod has been declining in the past few decades. We can model this decline with the equation \( y = -0.003x^2 + 0.120x + 1.909 \) where \( x \) is the number of years since 1950 and \( y \) is the catch in millions of metric tons. In which year did the catch reach its maximum? In which year will the catch reach 0? For what years was the catch increasing? Decreasing? When was the catch greater than 2.5 million metric tons? Show all of your work or explain how you came up with your solutions.

23. To determine the appropriate landing speed of an airplane, the formula \( D = 0.1x^2 - 3x + 22 \) is used, where \( x \) is the initial landing speed in feet per second and \( D \) is the distance needed in feet. If the landing speed is too fast, the pilot may run out of runway; if the speed is too slow, the plane may stall. What is the appropriate landing speed if the runway is 800 feet long? Show all of your work or explain how you came up with your solution.

24. The following table gives the number of motor vehicle thefts (in thousands) in the U.S. for the years 1983 - 1993. \( x = 1 \) represents 1983. Use the regression capabilities of your calculator to fit a cubic model to this data. Round to three decimal places. Use your model to predict the number of motor vehicle thefts in 1995.

<table>
<thead>
<tr>
<th>Year (x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
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</thead>
<tbody>
<tr>
<td>Thefts (y)</td>
<td>1008</td>
<td>1032</td>
<td>1103</td>
<td>1224</td>
<td>1289</td>
<td>1433</td>
<td>1565</td>
<td>1636</td>
<td>1662</td>
<td>1611</td>
<td>1561</td>
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25. A security panel with \( n \) buttons is disarmed when four different buttons are pushed in the correct order. The total number of four-button sequences in which no button is pushed more than one time is given by \( P(n) = n^4 - 6n^3 + 11n^2 - 6n \) where \( n \) is an integer greater than or equal to 4. For security reasons, the manufacturer of the panels requires that \( P(n) \geq 14,000 \). Find the least number of buttons that will meet the manufacturer’s requirement. Show all of your work or explain how you came up with your solution.

26. The town of Smallsville was founded in 1900. Its population \( y \) (in hundreds) is given by the function \( y = -0.1x^4 + 1.7x^3 - 9x^2 + 14.4x + 5 \) where \( x \) is the number of years since the foundation of Smallsville.

What was the population of Smallsville when it was founded? When did Smallsville become a "ghost town" with no one living in it? What was the largest population of Smallsville after 1905? When did Smallsville reach that population? Show all of your work or explain how you came up with your solution.

27. Pollution in the Tangipahoa River has been blamed primarily on the area dairy farmers. The formula \( C = \frac{400000p}{100 - p} \) is used to model the cost in dollars to area dairy farmers for removing \( p\% \) of the coliform bacteria from the river. If $1.2 million is spent, what percentage of the coliform bacteria has been removed? Show all of your work or explain how you came up with your solution.

28. A formula to determine the monthly payment (PMT) for a car loan, home mortgage, or other installment loan is given by \( \text{PMT} = P \left( \frac{1}{1 + \frac{i}{12}} \right)^n \) where \( P \) (called the present value) is the amount borrowed, \( i \) is the annual interest rate, and \( n \) is the total number of payments. John purchases a car and secures a loan for $9000 at an annual interest rate of 10\% for a term of 4 years. What is his monthly car payment? If he makes all 48 payments, how much money will he have paid? How much of this total is interest? Show all of your work or explain how you came up with your solution.
29. Sam Donahue has $8765 in his savings account on January 1. The account pays 5.25% interest compounded daily. On February 12, he deposited $936. Then on March 20, he deposited a tax refund check for $650. Find the balance at the end of the quarter and the interest earned during the quarter. Show all of your work or explain how you came up with your solution.

30. Astronomers use the distance modulus of a star as a method of determining the star’s distance from Earth. The formula is \( M = 5 \log r - 5 \), where \( M \) is the distance modulus and \( r \) is the star’s distance from Earth in parsecs (One parsec is approximately 3.3 light-years or \( 2.1 \times 10^{13} \) miles.) How many parsecs from Earth is a star with a distance modulus of 3? Show all of your work or explain how you came up with your solution.

31. Usually short-term interest rates (less than 2 years) are lower than long-term rates (more than 10 years) because short term securities are less risky than long-term ones. However, in periods of high inflation, the situation is reversed. During the early 1980s inflation was very high in the U.S. The rates for short- and long-term U.S. Treasury securities during 1980 are given in the following table. Use this data and the regression capabilities of your calculator to generate a logarithmic model for the data. Then use your model to predict the interest rate in 1980 for a 15 year T-bill. Show all of your work or explain how you came up with your solution.

<table>
<thead>
<tr>
<th>Term (years)</th>
<th>.5</th>
<th>1</th>
<th>5</th>
<th>10</th>
<th>20</th>
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<tbody>
<tr>
<td>Rate (%)</td>
<td>15</td>
<td>14</td>
<td>13.5</td>
<td>12.8</td>
<td>12.5</td>
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32. Industrial psychologist study employee training programs to assess the effectiveness of the instruction. In one study, the percent score \( P \) on a test for a person who has completed \( t \) hours of training was given by \( P = \frac{100}{1 + 30e^{-0.88t}} \). How many hours of training are necessary to achieve a 70% on the test? Show all of your work or explain how you came up with your solution.

33. Sodium-24 is a radioactive isotope of sodium that is used to study circulatory dysfunction. 4 micrograms of sodium-24 is injected into a patient and 5 hours later, 3.178 micrograms remain. Assuming that the isotope is dissipating exponentially, write an exponential decay model which gives the amount of sodium-24 (A) as a function of time \( t \) in hours. Use your model to determine the amount of isotope left after 10 hours. Show all of your work or explain how you came up with your solution.

34. Congratulations, you’ve just won $2500! You decide to invest in a mutual fund and save the money for an emergency. After 3 years, you have $2932.12. What was the interest rate on the fund if the interest was compounded quarterly? Show all of your work or explain how you came up with your solution.

35. Assuming that air resistance is proportional to the velocity of a falling object, the velocity (in feet per second) of an object \( t \) seconds after it has been dropped is given by \( V = 82\left(1 - e^{-39t}\right) \). When will the velocity be 70 feet per second? Show all of your work or explain how you came up with your solution.

36. A detective discovered a body in a vacant lot at 7 a.m. and found that the body temperature was 80°F. The county coroner examined the body at 8 a.m. and found that the body temperature was 72°. Assuming that the body temperature was 98° when the person died and that the air temperature was a constant 40° all night, what was the approximate time of death? Show all of your work or explain how you came up with your solution.

37. Theophylline, an asthma medicine, is to be prepared from an elixir with a drug concentration of 5 mg/mL and a cherry-flavored syrup that is to be added to hide the taste of the drug. How much of each must be used to prepare 100 milliliters of solution with a drug concentration of 2 mg/mL? Show all of your work or explain how you came up with your solution.

38. The Lincoln Park Zoo has different admission prices for adults and children. When Mr. and Mrs. Weaver went with their five children, the bill was $33. If Mrs. Wong and her three children got in for $18.50, then what is the price for an adult’s ticket and what is the price of a child’s ticket? Show all of your work or explain how you came up with your solution.
39. The employees from maintenance go for coffee together every day at 9 a.m. On Monday, Hector paid $5.45 for three cartons of milk, four cups of coffee, and seven doughnuts. On Tuesday, Guillermo paid $3.50 for four milks, two coffees, and eight doughnuts. On Wednesday, Anna paid $5.15 for two milks, five coffees, and six doughnuts. On Thursday, Alphonse had to pay for five milks, two coffees, and nine doughnuts. How much change did he get back for his $10 bill? Show all of your work or explain how you came up with your solution.

40. Farmer Bill is planning to raise wheat and barley. Each acre of wheat yields a profit of $50, and each acre of barley yields a profit of $70. To sow the crop, he arranges to borrow a tractor and a tiller from his neighbor. The tractor is available for 200 hours and the tiller is available for 100 hours. Sowing an acre of barley requires 3 hours of tractor time and 2 hours of tilling. Sowing an acre of wheat requires 4 hours of tractor time and 1 hour of tilling. How many acres of each crop should Bill plant to maximize his profit? Show all of your work or explain how you came up with your solution.

41. Dr. Pheblitz instructs his nurse to give Mr. Bennett .5 mg of Medex. The only form of Medex in stock is a liquid containing .125 mg per 4 mL. How much of this liquid should the nurse give Mr. Bennett? Show all of your work or explain how you came up with your solution.

42. A child weighing 33 lb is ordered amoxicillin oral suspension 150 mg by mouth every 8 hours. Is this a safe dose for a child of that weight? The label states that children should receive 20 to 40 mg/kg/day. Show all of your work or explain how you came up with your solution.

43. A machined block 3.345 inches thick must be reduced by 18%. What is the new thickness after the reduction? Show all of your work or explain how you came up with your solution.

44. A landscape architect plans a circular reflection pool for a park. The area regulations will not permit a pool any larger than 254.4696 square meters. What is the largest permissible diameter that may be used for this reflection pool? Show all of your work or explain how you came up with your solution.

45. Rebecca Thomas is paid $285 per week by the Art Lovers of the Lake, plus a 2% commission on all sales over $500. During a certain week, her total sales were $764. Find her gross earnings. Show all of your work or explain how you came up with your solution.

46. Suppose you can rent a $500 television set for $30 a month and after 24 months you own it. Is this a good idea, or would it be better to charge it on your credit card and pay off that credit card account at a rate of $30 a month? Show all of your work or explain how you came up with your solution.

47. The diameter of a bolt is .5 inch and the bolt has a tensile strength of 64,000 psi. How much force can the bolt withstand? Show all of your work or explain how you came up with your solution.

48. The nineteenth century English mathematician Augustus De Morgan once made the following statement: “I was \(x\) years old in the year \(2x\).” In what year was De Morgan born? Show all of your work or explain how you came up with your solution.

49. You just aced your first math test and you want to celebrate by filling your bathtub with champagne. Your bathtub has a rectangular base and vertical sides. It is 7 feet long, 3 feet wide, and 4 feet deep. How many quarts of champagne will you need to fill your bathtub to a foot below the top? Show all of your work or explain how you came up with your solution.

50. The price (per pound) of bananas is linearly related to the mean July temperature in Quiriguía, Guatemala. When the mean July temperature is 90°F, bananas cost $1.29 per pound. When the mean July temperature is 93°F, bananas cost $1.34 per pound. What would be the price per pound if the mean temperature was 85°F?
Answer Key
1. approximately 23 weeks
2. \( I = 7.5t + 115, \) $212.50 billion dollars
3. scatter plot; \( y = 289.8x + 1164.364; \) \( r = 0.9836096226; \) 5511; fairly confident because \( r \) is close to 1
4. answers may vary
5. 24.04 feet per second
6. 16.4 mg/l per hour; after 3 hours
7. increasing on \((0, 3), (6, 15);\) decreasing on \((3, 6)\) and \((30, 39);\) constant on \((15, 30)\)
8. .4 cm
9. \( g(x) = 16 - x; \) x-axis reflection & vertical shift up 16
10. \( A = 36\pi t^2 \)
11. \( w = 0.45x \)
12. \( t = \frac{P - 20}{4} = 2.5P - 50 \) gives the number of years since 1970 until the town’s population is \( P \) thousand
13. 250 yards long and 70 yards wide
14. 5 mph
15. 180 months or 15 years
16. 229.94 feet
17. Both dividing by a complex number and rationalizing a denominator involve multiplying both the numerator and the denominator by the conjugate of the denominator.
18. 3.83 seconds
19. \( T > 279.57 \) K
20. approximately .5 second
21. maximum radius 4.480 inches, minimum radius 4.432 inches
22. 1970 \((x = 20);\) 2002 \((x = 52);\) inc from 1959 to 1970, dec from 1970 till now; the catch was over 2.5 million metric tons from roughly 1955 till 1984
23. approximately 104.5 feet per second
24. \( y = -2.432x^3 + 37.317x^2 - 70.372x + 1043.788, 1092.421 \) thousand
25. 13
26. 5, May of 1908, 790, February of 1907
27. 5%
28. $228.26, $10,956.48, $1956.48
29. $10,472.80, $121.80
30. approximately 40 parsecs
31. \( y = 14.33759 - 0.62561 \ln x, \) where \( x \) is the term of the security and \( y \) is the interest rate, 12.642%
32. approximately 48 hours
33. \( A = 4e^{-0.0401t}, \) 2.525 micrograms
34. 5.35%
35. approximately 4.9 seconds
36. 5:20 a.m.
37. 25 mL of elixir and 75 ml of syrup
38. $6.50 for an adult, $4 for a child
39. $3.95
40. 20 acres of wheat and 40 acres of barley for a maximum profit of $3800
41. 16 mL
42. The ordered dose of 450 mg in 24 hours is within the 300 - 600 mg/day safe range for a child weighing 33 lb.
43. 2.7429 inches
44. 18 meters
45. $290.28
46. The annual interest rate of the rent-to own option is about 40% and the nominal rate is 22%. If your credit card has an annual percentage rate lower than 22%, it would be best to use your credit card.
47. approximately 12,566 lbs
48. 1806
49. approximately 1885 quarts
50. $1.21