**Finite Math Departmental Review Part 1: 3.1 – 5.3**

**Directions:** Show all of your work on every problem.

*For problems 1 – 6, solve the linear programming problems graphically.*

1. **Minimize**

   \[ P = 30x + 15y \]

   **subject to:**
   
   \[ 2x + 3y \geq 30 \]
   
   \[ 2x + y \leq 26 \]
   
   \[ -6x + 5y \leq 50 \]
   
   \[ x \geq 0, y \geq 0 \]

2. **Maximize**

   \[ P = 4x + 4y \]

   **subject to:**
   
   \[ 2x + y \leq 20 \]
   
   \[ x + 2y \leq 16 \]
   
   \[ x \geq 0, y \geq 0 \]

3. **Maximize**

   \[ P = 40x + 60y \]

   **subject to:**
   
   \[ 2x + 3y \geq 30 \]
   
   \[ 2x+y \leq 26 \]
   
   \[ -x + y \leq 5 \]
   
   \[ x \geq 0, y \geq 0 \]

4. Bruce’s Sand & Gravel Company hauls gravel to a construction site, using a small truck and large truck. The carrying capacity and operating cost per load are given in the accompanying table. Bruce must deliver a minimum of 630 cubic yards per day to satisfy his contract with the builder. The union contract with his drivers requires that the total number of loads per day is a minimum of 10. How many loads should be made in each truck per day to minimize the total cost?

<table>
<thead>
<tr>
<th>Small Truck</th>
<th>Large Truck</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity (yd³)</td>
<td>70</td>
</tr>
<tr>
<td>Cost per Load</td>
<td>$97</td>
</tr>
</tbody>
</table>

5. A manufacturing company makes two types of water skis, a trick ski and a slalom ski. The relevant manufacturing data are given in the table.

<table>
<thead>
<tr>
<th>Department</th>
<th>Labor-Hours per Ski</th>
<th>Maximum Labor-Hours Available per Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fabricating</td>
<td>Trick Ski 6 Slalom Ski 4</td>
<td>204</td>
</tr>
<tr>
<td>Finishing</td>
<td>1 1</td>
<td>36</td>
</tr>
</tbody>
</table>

   a. If the profit on a trick ski is $30 and the profit on a slalom ski is $60, how many of each type of ski should be manufactured each day to realize a maximum profit? What is the maximum profit?

   b. If the profit on a slalom ski decreases to $45 how many of each ski should be manufactured each day to realize a maximum profit? What is the maximum profit?

6. Mr. Smith decides to feed his pet Doberman pinscher a combination of two dog foods. Each can of brand A contains 4 units of protein, 1 unit of carbohydrates, and 2 units of fat and costs 60 cents. Each can of brand B contains 1 unit of protein, 1 unit of carbohydrates, and 4 units of fat and costs 50 cents. Mr. Smith feels that each day his dog should have at least 7 units of protein, 4 units of carbohydrates, and 12 units of fat. How many cans of each dog food should he give to his dog each day to provide the minimum requirements at the least cost?

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8. Maximize \( z = 2x_1 + 3x_2 \)
subject to: 
\[ 5x_1 + x_2 \leq 60 \]
\[ 3x_1 + 2x_2 \leq 80 \]
\[ x_1 + x_2 \leq 70 \]
\[ x_1 \geq 0, \ x_2 \geq 0 \]

9. Maximize \( z = 7x_1 + 2x_2 + x_3 \)
subject to: 
\[ x_1 + 5x_2 + 5x_3 \leq 115 \]
\[ x_1 + 3x_2 + 10x_3 \leq 239 \]
\[ x_1 \geq 0, \ x_2 \geq 0, \ x_3 \geq 0 \]

10. Maximize \( z = x_1 + 3x_2 + x_3 + 7x_4 \)
subject to: 
\[ x_1 + 4x_2 + x_3 + x_4 \leq 47 \]
\[ 2x_1 + x_2 + 3x_4 \leq 98 \]
\[ x_1 \geq 0, \ x_2 \geq 0, \ x_3 \geq 0, \ x_4 \geq 0 \]

11. Jayanta is raising money for the homeless, and discovers each church group requires 2 hr of letter writing and 1 hr of follow-up calls, while each labor union needs 2 hr of letter writing and 3 hr of follow-up. She can raise $100 from each church group and $200 from each union. She has a maximum of 12 hours of letter writing and 14 hours of follow-up available each month. Determine the most profitable mixture of groups she should contact and the most money she can raise in a month.

12. A biologist has 600 kg of nutrient A, 800 kg of nutrient B, and 200 kg of nutrient C. These nutrients will be used to make 4 types of food, whose contents (in percent per kilogram of food) and whose “growth values” are shown in the table below. How many kilograms of each food should be produced in order to maximize total growth?

<table>
<thead>
<tr>
<th></th>
<th>P</th>
<th>Q</th>
<th>R</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>12.5</td>
<td>37.5</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>50</td>
<td>50</td>
<td>62.5</td>
</tr>
<tr>
<td>C</td>
<td>100</td>
<td>50</td>
<td>37.5</td>
<td>0</td>
</tr>
<tr>
<td>Growth Value</td>
<td>110</td>
<td>100</td>
<td>80</td>
<td>70</td>
</tr>
</tbody>
</table>

13. A department store chain has up to $22,000 to spend on television advertising for a sale. All ads will be placed with one television station, where a 30-secnd ad costs $1,000 on daytime TV and is viewed by 14,000 potential customers, $2,000 on prime-time TV and is viewed by 24,000 potential customers, and $1500 on late night TV and is viewed by 18,000 potential customers. The television station will not accept a total of more than 15 ads in all three time periods. How many ads should be placed in each time period in order to maximize the number of potential customers who will see the ads? How many potential customers will see the ads?

**On problems 14 – 17, use the simplex method to solve the nonstandard type problems.**

14. Maximize \( P = 12x + 10y \)
subject to: 
\[ x + 5y \geq 27 \]
\[ x + y \leq 46 \]
\[ x \geq 0, \ y \geq 0 \]

15. Minimize \( P = 48y_1 + 30y_2 + 59y_3 \)
subject to: 
\[ 6y_1 + 3y_2 + 7y_3 \leq 12 \]
\[ 6y_1 + 6y_2 + 5y_3 \geq 10 \]
\[ y_1 \geq 0, \ y_2 \geq 0, \ y_3 \geq 0 \]
16. A company’s lawn seed mixtures contain three types of seeds: bluegrass, rye, and Bermuda. The costs per pound of the three types are 13¢, 18¢, and 7¢, respectively. In each batch there must be at least 25% bluegrass seed and the amount of Bermuda must be no more than two-thirds the amount of rye. To fill current orders the company must make at least 3000 pounds of the mixture. How much of each kind of seed should be used to minimize cost?

4.4.26

17. A company is developing a new additive for gasoline. The additive is a mixture of three liquids, I, II, and III. For proper performance, the total amount of additive must be at least 6 oz per gal of gasoline. However, for safety reasons, the amount of additive should not exceed 18 oz per gal of gasoline. At least 1/6 oz of liquid I must be used for every ounce of liquid II, and at least 1 oz of liquid III must be used for every ounce of liquid I. The costs of I, II, and III are $0.40, $0.24, and $0.64 per oz, respectively. What is the mixture of the three liquids that produces the minimum cost of the additive? How much of the additive should be used per gallon of gasoline?

4.4.31

Note: The problems from section 5.1 that require you to solve for time or rate may be solved either algebraically or graphically whichever method your instructor requires.

18. Find the simple interest for a principal of $2000 invested at a rate of 4% for 3 months. Round to the nearest cent.

5.1.5

19. Use the compound interest formula to compute the total amount after $2500 is invested at 8% over 10 years compounded semiannually. Then compute the interest earned.

5.1.21

20. A family plans to retire in 10 years and expects to need $600,000. Determine how much they must invest today at 7.5% compounded annually to accomplish their goal.

5.1.25

21. Determine the effective rate for $1 invested for 1 year at 4.6% compounded quarterly.

5.1.33

22. An account invested in a money market fund grew from $73,333.17 to $73,623.53 in a month. What was the interest rate to the nearest tenth of a percent?

5.1.39

23. As the prize in a contest, you are offered $22,000 now or $27,000 in 5 years. If the money can be invested at 7% compounded annually, which prize will be worth more in 5 years and by how much? Round answers to the nearest cent.

5.1.49

24. At the average annual inflation rate of 4.1%, about how long would it take for the general level of prices in the economy to double? Round to the nearest tenth.

5.1.54

25. A gift of 49000 to a city grew to $10,000,000 in 200 years. At what interest rate compounded annually would this growth occur? Round to the nearest hundredth of a percent.

5.1.60

26. An amount of $1000 was deposited in a bank at a rate of 7% compounded quarterly for 3 years. The rate then increased to 9% and was compounded quarterly for the next 3 years. If no money was withdrawn, what was the balance at the end of this time? Round to the nearest cent.

5.1.62

27. Find the future value of the ordinary annuity. Interest is compounded annually. (Round to the nearest cent) $R = 3000, i = 0.09, n = 10$

5.2.21

28. Find the future value of an ordinary annuity if payments are made in the amount R and interest is compounded as given. Then determine how much of this value is from contributions and how much is from interest. (Round to the nearest cent) $R = 9,000, 8% interest compounded semiannually for 3 years.$

5.2.25

29. Find the future value of the following annuity due. Assume that interest is compounded annually, there are n payments of R dollars and the interest rate is i. $R=800, i = 0.04, and n=8$ (Round to the nearest cent)
30. Find the future value of the following annuity due. Then determine how much of this value is from contributions and how much is from interest. $1050 is deposited at the beginning of each semiannual period for 6 years at 7.18% compounded semiannually. Round to the nearest cent as needed. 5.2.37

31. Find the periodic payment which will amount to a sum of $12000 if an interest rate 8% is compounded annually at the end of 10 consecutive years. Round to the nearest cent. 5.2.39

32. Find the size of each of 7 payments made at the end of each year into a 6% rate sinking fund which produces $41000 at the end of 7 years. Round to the nearest cent. 5.2.43

33. In order to accumulate enough money for a down payment on a house, a couple deposits $301 per month into an account paying 6% compounded monthly. If payments are made at the end of each period, how much money will be in the account after 3 years? Round to the nearest cent. 5.2.51

34. A father opened a savings account for his daughter on the day she was born, depositing $800. Each year on her birthday, he deposits another $800, making the last deposit on her 17th birthday. If the account pays 5% interest, compounded annually, how much is in the account at the end of the day on his daughter’s 17th birthday. How much interest has been earned? Round your answers to the nearest cent. 5.2.52

35. Starting at age 50, a woman puts $1300 at the end of each quarter into a retirement account that pays 7% interest compounded quarterly. When she reaches age 60, she withdraws the entire amount and places it in a mutual fund account that pays 9% compounded monthly. From then on she deposits $200 in a mutual fund at the end of each month. How much is in the account when she reaches age 65? Round your final answer to the nearest dollar after rounding intermediate steps to the nearest cent. 5.2.54

36. Ingrid wants to buy a $20,000 car in 5 years. How much money must she deposit at the end of each quarter in an account paying 5.5% compounded quarterly so that she will have enough to pay for her car? Round to the nearest cent. 5.2.57

37. Find the accumulated value of an IRA account into which $750 is invested at the end of each quarter at an interest rate of 2.0% compounded quarterly if you start the IRA at age 25 and retire at age 60. Round to the nearest cent. 5.2.61

38. Hank made payments of $200 per month at the end of each month for 30 years to purchase a piece of property. He promptly sold it for $174,362. What annual interest rate would he need to earn on an ordinary annuity for a comparable rate of return? Round to the nearest hundredth of a percent. 5.2.65

39. Find the present value of an ordinary annuity with deposits of $20,799 semiannually for 8 years at 8.8% compounded semiannually. Round to the nearest cent. 5.3.11

40. Find the lump sum deposited today that will yield the same total amount as payments of $13,000 at the end of each year for 10 years, at an interest rate of 5% compounded annually. Round to the nearest cent. 5.3.15

41. Find the payment necessary to amortize a 5.3% loan of $7700 compounded annually, with 6 annual payments. Calculate the total payments and total amount of interest paid. Round your answers to the nearest cent. 5.3.21

42. Find the monthly house payments necessary to amortize a 4.8% loan of $160,900 over 15 years. Round to the nearest cent. 5.3.29

43. A new employee charged $9360 on his credit card to relocate for his first job. After noticing that the interest rate for his balance was 18% compounded monthly, he stopped charging on that account. He wishes to pay off his balance in 2 years using automatic payments sent at the end of each month.
   a. What monthly payment must he make to pay off the account at the end of 2 years?
   b. How much total interest will he have paid?
44. When Albertina opened her law office, she bought $17,000 worth of law books and $6,800 worth of furniture. She paid $1,300 down and agreed to amortize the balance with semiannual payments for 5 years at 12% compounded semiannually. Find the amount of each payment. How much of the first payment was interest? Round to the nearest cent.