Finite Math Exam 1 Review: Chapters 3, 4, 5

General Guidelines:

Do not try to cram. It may work for other subjects, but it is does not work for math. Begin studying at least one week in advance. Read back through your notes. Look over all examples worked in class. Rework problems collected for homework checks. Get clarification on any areas you are unclear about. Work as many of the suggested review problems as you possibly can. Use these problems to create practice exams. Work your practice exams on blank paper with no resources other than a calculator. Your book, notebook, homework, solutions manual, etc. should be closed. Get a good night's sleep the night before the exam. Do not sacrifice sleep to cram, it is self-defeating. Look over definitions, formulas, and algorithms right before you go to sleep and again when you wake up. Eat a little protein before you come to take the exam and be sure you are well hydrated. Schedule lots of time to take the exam. Don't rush yourself. As soon as you get your exam, do a "brain-dump".

Directions: Show all of your work on all problems. Give exact solutions when possible, otherwise round monetary values to two decimal places and other answers to three decimal places.

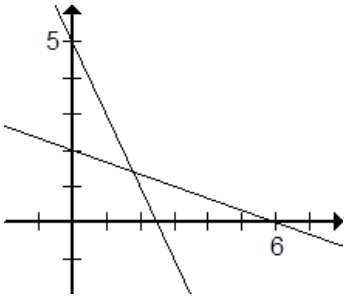
Part One

1. Graph the feasible region for the following system of inequalities. Show all of your work, including a t-chart for each oblique border with the x-intercept, y-intercept, and one additional point. Also show your shading tests and their results. [3.1]

   $-3x + y \le 5 \quad x - 2y \le 4 \quad x \ge 0 \quad y \ge 0$

2. Complete the following linear programming problem by a) Shading the feasible region. b) Finding the corner points. c) Evaluating C at each corner point and finding the minimum. You do not have to include t-charts for the border lines. [3.2]

   Minimize $C = 4x + 8y$ subject to $2x + y \le 5 \quad x + 3y \ge 6 \quad x \ge 0 \quad y \ge 0$



3. Solve the following linear programming problems graphically. For each oblique border, include a t-chart with the x-intercept, the y-intercept, and one additional point, along with your shading test and its results. Clearly graph the feasible region, find the corner points, list the solutions of the objective function, and state your final answer. [3.2]

a. Maximize $P = 4x + 5y$ subject to $2x + y \le 50 \quad x + 3y \le 75 \quad x \ge 0 \quad y \ge 0$

b. Minimize $C = 12x + 10y$ subject to $4x + y \ge 40 \quad 2x + 3y \ge 60 \quad x \ge 0 \quad y \ge 0$

4. For each of the following linear programming problems: Label the variables and write the objective function and constraints. Solve the problem graphically. For each oblique border include a t-chart with the x-intercept, the y-intercept, and one additional point, along with your shading test and its results. Clearly graph the feasible region, find the corner points, list the solutions of the objective function, and state your final answer. [3.3]

a. An office manager needs to purchase new filing cabinets. He knows that Ace cabinets cost \$80 each, requires 6 ft^2 of floor space, and hold 24 ft^3 of files. On the other hand, each Excello cabinet costs \$160,

requires 8 ft^2 of floor space, and holds 36 ft^3. His budget permits him to spend no more than $1120 on cabinets, while the office has space for no more than 72 ft^2 of cabinets. The manager desires the greatest storage capacity within the limits imposed by funds and space. How many of each type of cabinet should he buy?

b. A 4-H member raises only goats and pigs. She wants to raise no more than 16 animals, including no more than 10 goats. She spends $25 to raise a goat and $75 to raise a pig, and she has $900 available for this project. The 4-H member wishes to maximize her profits. Each goat produces $12 in profit and each pig $40 in profit. How many of each animal should she raise to maximize her profit?

5. For the adjacent standard maximization problem do the following. [4.1]

a) Use a slack variable to convert the structural constraints into linear equations.
b) Rewrite the objective function.
c) Write the initial simplex tableau.
d) Write the solution from the initial tableau.

Maximize	$z = 16x_1 + 14x_2 + 11x_3$
subject to:	$3x_1 + 2x_2 + x_3 \le 8$
	$5x_1 + 3x_2 - 2x_3 \le 15$
with	$x_1 \ge 0 \quad x_2 \ge 0 \quad x_3 \ge 0$

6. Label the variables and write the objective function and constraints for the following linear programming problems. Then add slack variables, rewrite the objective function, and write the initial simplex tableau. You do <u>not</u> have to solve the problems, just set them up. [4.1]

a. A manufacturer makes two grades of concrete. Each bag of the high-grade concrete contains 10 kilograms of gravel and 5 kilograms of cement, while each bag of low-grade concrete contains 12 kilograms of gravel and 3 kilograms of cement. There are 1,920 kilograms of gravel and 780 kilograms of cement currently available. The manufacturer can make a profit of $1.20 on each bag of the high-grade and $1.00 on each bag of the low-grade concrete. How many bags of each grade should be made up from available supplies to generate the maximum profit? What is the maximum profit?

b. A fully automated plastics factory produces two toys, a racing car and a jet airplane, in three stages: molding, painting, and packaging. After allowing for routine maintenance, the equipment for each stage can operate no more than 150 hours per week. Each batch of racing cars requires 6 hours of molding, 2.5 hours of painting, and 5 hours of packaging while each batch of jet airplanes requires 3 hours of molding, 7.5 hours of painting, and 5 hours of packaging. If the cost per batch of toys is $120 for cars and $100 for airplanes, how many batches of each toy should be produced each week to minimize cost? What is the minimum cost?

7. Solve the following standard linear programming problems by using the simplex method. Record the slack equations and rewritten objective function, as well as the matrix and row operations from every step. Circle the pivot in each matrix. [4.2]

a. Maximize $f = 14x + 10y + 12z$ subject to $2x + 2y + 8z \le 40 \quad 4x - 5y + 6z \le 60 \quad 2x - 2y + 6z \le 24$ with $x \ge 0 \quad y \ge 0 \quad z \ge 0$

b. Maximize $f = 2x + 2y + z$ subject to $2x + y + 2z \le 14 \quad 2x + 4y + z \le 26 \quad x + 2y + 3z \le 28$ with $x \ge 0 \quad y \ge 0 \quad z \ge 0$

c. Kane Manufacturing has a division that produces two models of hibachis, model A and model B. To produce each model A hibachi requires 3 pounds of cast iron and 6 minutes of labor. To produce each model B requires 4 pounds of cast iron and 3 minutes of labor. The profit for each model A is $2, and the profit for each model B is $1.50. If 1000 pounds of cast iron and 20 labor-hours are available for the production of hibachis each day, how many hibachis of each model should the division produce to maximize Kane's profits? What is the largest profit the company can realize? Is there any raw material left over?

8. Solve the following nonstandard linear programming problems. Record the setup, as well as the matrix and row operations from every step. Circle the pivot in each matrix. [4.4]

a. Maximize $P = x + 2y$ subject to $4x + 3y \le 18$ $-x + 3y \ge 3$ with $x \ge 0$ $y \ge 0$

b. Minimize $C = 2x + 3y$ subject to $x + y \le 5$ $x + 3y \ge 9$ $-2x + y \le 2$ with $x \ge 0$ $y \ge 0$

Part Two

For all financial problems, label all variables (P = $3250.47, r = .034, t = 7.5, m = 12, etc.); show your setup (the appropriate formula with the values of the variables plugged in); and the final solution. You do not need to show all of your arithmetic.

Find the simple interest. Assume a 360–day year. Round results to the nearest cent.

1. $3180 at 7% for 9 months

2. $25,210 at 4.75% for 112 days

Find the compound amount for the deposit. Round to the nearest cent.

3. $8940 at 4.8% compounded semiannually for 12 years

Find the amount that should be invested now to accumulate the following amount, if the money is compounded as indicated.

4. $4200 at 4.2% compounded quarterly for 8 years

Find the effective rate corresponding to the given nominal rate. Round results to the nearest 0.01 percentage points.

5. 5% compounded quarterly

Solve the problem.

6. How long will it take for prices in the economy to double at a 8% annual inflation rate? Round answer to the nearest year.

7. Barbara knows that she will need to buy a new car in 3 years. The car will cost $15,000 by then. How much should she invest now at 8%, compounded quarterly, so that she will have enough to buy a new car?

Find the future value of the ordinary annuity. Interest is compounded annually, unless otherwise indicated.

8. R = $2,500, i = 8% interest compounded quarterly for 16 years

Find the future value of the annuity due. Assume that interest is compounded annually, unless otherwise indicated.

9. $200 deposited at the beginning of each quarter for 15 years at 8.4% compounded quarterly

Find the periodic payment that will render the sum.

10. S = $60,000, interest is 4% compounded annually, payments made at the end of each year for 5 years

Find the amount of each payment to be made into a sinking fund so that enough will be present to accumulate the following amount. Payments are made at the end of each period. The interest rate given is per period.

11. $80,000; money earns 2.3% compounded monthly for $1\frac{5}{12}$ years

Solve the problem. Round to the nearest cent.

12. $765.13 is deposited at the end of each month for 4 years in an account paying 12% interest compounded monthly. Find the amount of the account.

Solve the problem.

13. Which of the following investments is larger?
    A) $500 is deposited monthly for 20 years and earns 4.25% interest compounded monthly.
    B) $15,000 is deposited annually for 10 years and earns 4.25% interest compounded annually.

14. If $600,000 is to be saved over 10 years, how much should be deposited annually if the investment earns 9.5% interest compounded annually?
    A) $53,427.37 B) $38,559.69 C) $33,262.16 D) $45,122.73

Solve the problem. Round to the nearest cent.

15. Larry wants to start an IRA that will have $410,000 in it when he retires in 24 years. How much should he invest semiannually in his IRA to do this if the interest is 5% compounded semiannually?

Find the present value of the ordinary annuity.

16. Payments of $73 made quarterly for 10 years at 8% compounded quarterly

Find the lump sum deposited today that will yield the same total amount as this yearly payment (made at the end of each year for 20 years at the given interest rate, compounded annually.

17. $2300 at 6%

Find the payment necessary to amortize the loan.

18. $13,500; 12% compounded monthly; 48 monthly payments

Find the monthly house payment necessary to amortize the following loan.

19. $125,000 at 9.6% for 30 years

Solve the problem.

20. Tasha borrowed $11,000 to purchase a new car at an annual interest rate of 7.2%. She is to pay it back in equal monthly payments over a 4 year period. How much total interest will be paid over the period of the loan? Round to the nearest dollar.

21. You have a $4312 credit card debt, and you plan to pay it off through monthly payments of $75. If you are being charged 18% interest per year, how long (to the nearest tenth of a year) will it take you to repay your debt?

22. Cara has a loan from her credit union at a rate of 8.5% for which her payments are $185 per month. The interest is computed on a daily basis on the unpaid balance of the loan. If the loan balance after her last payment was $2387 and Cara makes her next payment 30 days later, how much of the payment is paid toward interest?

Prepare an amortization schedule showing the first four payments for the loan.

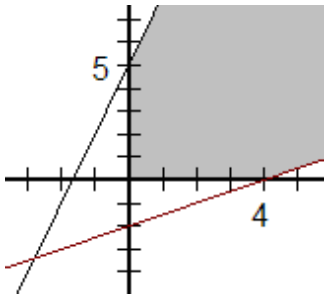
23. Mary finances $150,000 towards the purchase of a new home through a 20–year mortgage. The interest rate applied to the monthly unpaid balance is 5.2%

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
1				
2				
3				
4				

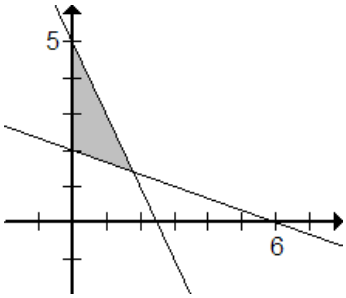
Answers

Part One

1.



2.



Corner Point	C = 4x + 8y
(0, 2)	C = 16
(0, 5)	C = 40
(1.8, 1.4)	C = 18.4

Minimum C = 16 when x = 0 and y = 2

3a. The maximum value of P is 160 and it occurs when x = 15 and y = 20.

3b. The minimum value of C is 232 at it occurs when x = 6 and y = 16.

4a. x = # of number Ace cabinets. y = # of Excello cabinets.
Maximize capacity, C = 24x + 36y, subject to $80x + 160y \le 1120$ $6x + 8y \le 72$ $x \ge 0$ $y \ge 0$
The office manager should by 8 Ace cabinets and 3 Excello cabinets to obtain the maximum storage capacity of 300 ft^3.

4b. x = # of goats. y = # of pigs
Maximize Profit, P = 12x + 40y, subject to $x + y \le 16$ $x + 3y \le 36$ $x \le 10$ $x \ge 0$ $y \ge 0$
She should raise 12 pigs and no goats to maximize her profit to $480.

5a. $3x_1 + 2x_2 + x_3 + s_1 = 8$

$5x_1 + 3x_2 - 2x_3 + s_2 = 15$

b. $-16x_1 - 14x_2 - 11x_3 + z = 0$

c.
$$\left[\begin{array}{cccccc|c} x_1 & x_2 & x_3 & s_1 & s_2 & z & \\ 3 & 2 & 1 & 1 & 0 & 0 & 8 \\ 5 & 3 & -2 & 0 & 1 & 0 & 15 \\ \hline -16 & -14 & -11 & 0 & 0 & 1 & 0 \end{array}\right]$$

d. $z = 0, x_1 = 0, x_2 = 0, x_3 = 0, s_1 = 8, s_2 = 15$

6a. x = # of bags of high-grade cement. y = # of bags of low-grade cement

Maximize profit, P = 1.2x + y, subject to the following constraints:

$10x + 12y \le 1{,}920$ $5x + 3y \le 780$ $x \ge 0$ $y \ge 0$

$10x + 12y + s_1 = 1920$

$5x + 3y + s_2 = 780$

$-1.2x - y + P = 0$

$$\left[\begin{array}{ccccc|c} x & y & s_1 & s_2 & P & \\ 10 & 12 & 1 & 0 & 0 & 1920 \\ 5 & 3 & 0 & 1 & 0 & 780 \\ \hline -1.2 & -1 & 0 & 0 & 1 & 0 \end{array}\right]$$

6b. x = # of batches of cars. y = # of batches of jets

Minimize cost, C = 120x + 100y, subject to the following constraints:

$6x + 3y \le 150$ $2.5x + 7.5y \le 150$ $5x + 5y \le 150$ $x \ge 0$ $y \ge 0$

$$\begin{aligned} 6x + 3y + s_1 &= 150 \\ 2.5x + 7.5y + s_2 &= 150 \\ 5x + 5y + s_3 &= 150 \\ -120x - 100y + C &= 0 \end{aligned}$$

$$\left[\begin{array}{cccccc|c} 6 & 3 & 1 & 0 & 0 & 0 & 150 \\ 2.5 & 7.5 & 0 & 1 & 0 & 0 & 150 \\ 5 & 5 & 0 & 0 & 1 & 0 & 150 \\ \hline -120 & -100 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

7a. Maximum f is 264 when x = 16, y = 4, z = 0, $s_1 = 0$, $s_2 = 16$, and $s_3 = 0$

7b. Maximum f is 18 when x = 5, y = 4, z = 0, $s_1 = 0$, $s_2 = 0$, and $s_3 = 15$

7c. x = # model A hibachis. y = # model B hibachis
Maximize profit, P = 2x + 1.5y, subject to $3x + 4y \le 1000$ $6x + 3y \le 1200$ $x \ge 0$ $y \ge 0$
They should produce 120 model A hibachis and 160 model B hibachis to earn a maximum profit of $480. There will be no raw material left over.

8a. Maximum P = 12 when x = 0, y = 6, $s_1 = 0$, $s_2 = 15$

8b. Minimum C = $\frac{66}{7}$ when x = $\frac{3}{7}$, y = $\frac{20}{7}$, s_1 = $\frac{12}{7}$, $s_2 = 0$, and $s_3 = 0$

Part Two

1. $166.95
2. $372.55
3. $15,795.61
4. $3006.68
5. 5.09%
6. 9 years
7. $11,827.40
8. $318,936.66
9. $24,112.34
10. $11,077.63
11. $4634.14
12. $46,843.25
13. A
14. B
15. $4512.46
16. $1996.95
17. $26,380.82
18. $355.51
19. $1060.20
20. $1693
21. 11.1 years
22. $16.68
23.

Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
1	1006.58	650.00	356.58	149,643.42
2	1006.58	648.45	358.13	149,285.29
3	1006.58	646.90	359.68	148,925.61
4	1006.58	645.34	361.24	148,564.37