5.1 Simple and Compound Interest

I. Simple Interest

A. Introduction

Simple Interest is interest paid (or earned) only on the amount borrowed (or invested), and not on past interest. It is usually used for loans or investments of a year or less.

B. Definitions

I = Interest = the fee paid to borrow money or the income earned for loaning money

P = Principal or Present Balance = the money being borrowed or loaned

r = Rate = the annual interest rate; usually given as a percentage and expressed as a decimal; To convert between percentage and decimal form, move the decimal two places to the left. Examples: 4% = .04; 5.6% = .056; 3.75% = .0375

t = Time in years; 1 year = 12 months = 52 weeks = 360 days (ordinary interest) or 365 days (exact interest); Examples: 5 months $\rightarrow 12 \frac{5}{12}$; 11 weeks $\rightarrow t = 11 \frac{4}{52}$

A = Accumulated Balance or Future Value or Maturity Value = the sum of the principal and the interest

C. Formulas

Interest Formula:  
$$I = P \cdot r \cdot t$$

Future Value Formula:  
$$A = P(1 + r \cdot t) \quad \text{or} \quad A = P + Prt \quad \text{or} \quad A = P + I$$

Present Value Formula:  
$$P = \frac{A}{1 + rt}$$

Example 1

Find the simple interest.

a. $1974 at 6.3\% for 25 weeks \[7\]

$$I = 1974 \times 0.063 \times \frac{25}{52} = \$59.79$$

b. $7236.15 at 4.25\% for 30 days (Assume a 360-day year) \[10\]

$$I = 7236.15 \times 0.0425 \times \frac{30}{360} = \$25.63$$

Note: Round monetary values to the nearest cent.

Example 2

Find the maturity value and the amount of simple interest earned. \[11\]

$3125 at 2.85\% for 7 months

$$A = 3125 (1 + (0.0285 \times \frac{7}{12})) = \$3176.95$$

$$I = A - P = 3176.95 - 3125 = \$51.95$$

Example 3

A $1500 certificate of deposit held for 75 days was worth $1521.25. To the nearest tenth of a percent, what was the interest rate earned? Assume a 360-day year. \[39\]

$$I = 1521.25 - 1500 = \$21.25$$

$$r = \frac{I}{P \cdot t} = \frac{21.25}{(1500 \cdot \frac{75}{360})} = 0.068 \rightarrow 6.8\%$$
II. **Compound Interest**

A. **Introduction**

*Compound Interest* is paid (or earned) on both the principal and the interest.

B. **Definitions**

- \( m \) = number of times interest is compounded in one year;
  - annually: \( m = 1 \);
  - semi-annually: \( m = 2 \);
  - quarterly: \( m = 4 \);
  - monthly: \( m = 12 \);
  - weekly: \( m = 52 \);
  - daily: \( m = 360 \) (*ordinary interest*) or \( m = 365 \) (*exact interest*)

- \( i \) = interest rate per period; \( i = \frac{r}{m} \)

- \( n \) = total number of compounding periods; \( n = m \cdot t \)

C. **Formulas**

**Future Value Formula:**

\[
A = P \left(1 + \frac{r}{m}\right)^{(m \cdot t)} \quad \text{or} \quad A = P(1 + i)^n
\]

**Present Value Formula:**

\[
P = \frac{A}{\left(1 + \frac{r}{m}\right)^{(m \cdot t)}} \quad \text{or} \quad P = A \left(1 + \frac{r}{m}\right)^{-(m \cdot t)}
\]

Here \( P \) is the amount that should be deposited today to produce \( A \) dollars in \( t \) years.

**Example 4**

Find the compound amount for each deposit and the amount of interest earned.

- $9100 at 6.4% compounded quarterly for 9 years

\[
A = 9100 \left(1 + \frac{0.064}{4}\right)^{(4 \cdot 9)} = \$16,114.43 \quad \text{I} = A - P = 16,114.43 - 9100 = \$7,014.43
\]

**Example 5**

Find the present value (the amount that should be invested now to accumulate the following amount) if the money is compounded as indicated.

- $2,000 at 7% compounded semiannually for 8 years

\[
P = 2000 \left(1 + \frac{0.07}{2}\right)^{(2 \cdot 8)} = \$1153.41
\]

**Recall:** From algebra we know if \( A = B^n \), then \( \log A = \log B^n \).

In addition, the Power Rule of Logarithms states \( \log B^n = n \cdot \log B \).

Thus, \( A = B^n \) \( \rightarrow \) \( \log A = \log B^n \) \( \rightarrow \) \( \log A = n \cdot \log B \) \( \rightarrow \) \( n = \frac{\log A}{\log B} \)

**Example 6**

The consumption of electricity has increased historically at 6% per year. If it continues to increase at this rate indefinitely, find the number of years before the electric utilities will need to double their generating capacity.

Let \( P \) = current generating capacity, then double this capacity will be \( 2P \).
\[ 2P = P\left(1 + \frac{.06}{1}\right)^{(1t)} \rightarrow \frac{2P}{P} = \frac{P(1.06)^t}{P} \rightarrow 2 = 1.06^t \rightarrow \log(2) = \log(1.06^t) \rightarrow \]

\[ \log(2) = t \cdot \log(1.06) \rightarrow t = \frac{\log(2)}{\log(1.06)} \rightarrow t = 11.896 \approx 12 \text{ years} \]

III. Effective Rate

If $1 is deposited at 5% compounded monthly, at the end of one year the value of the deposit is $1.0512, an increase of 5.12% over the original $1. The actual increase of 5.12% is somewhat higher than the stated increase of 5%. To differentiate between these two numbers, 5% is called the **stated or nominal rate** of interest and 5.12% is called the **effective rate** \( r_e \) or **annual percentage rate (APR)**.

\[ r_e = \left(1 + \frac{r}{m}\right)^m - 1 \]

**Example 7**

Find the effective rate corresponding to a nominal rate of 7.25% compounded semiannually. \[ [35] \]

\[ r_e = \left(1 + \frac{.0725}{2}\right)^2 - 1 = .0738 \rightarrow r_e \approx 7.38\% \]