7.2 Applications of Venn Diagrams

I. Venn Diagram with One Set

One Set Leads to Two Regions:

\[ 1 = A' \quad ; \quad 2 = A \]

II. Venn Diagram with Two Sets

Two Sets Lead to Four Regions

III. Venn Diagram with Three Sets

Three Sets Lead to Eight Regions

ex: #3

\[ A' \quad B \quad \text{All elements not in A} \]

\[ A' \cup B \quad \text{All elements in B} \]

#7

\[ S \quad \text{All elements not in S} \]

\[ S' = \emptyset \]

#12

\[ S \quad \text{All elements in A and not in C} \]

\[ S \cup C' \quad \text{All elements in C} \]

\[ (A \cap C') \cup B \quad \text{All elements in B} \]

IV. Union Rule for Sets (Blue Box p. 357)

\[ n(A \cup B) = n(A) + n(B) - n(A \cap B) \]

Alternate Form:

\[ n(\overline{A \cup B}) = n(A) + n(B) - n(A \cup B) \]

\[ \text{indicates number of elements in the set} \]

\[ \text{See problem on top of p. 61} \]

ex: #42

\[ n(A) = 15 \quad n(B) = 30 \quad n(A \cup B) = 33 \quad n(A \cap B) = 15 + 30 - 33 = 12 \]
EX: If \( n(A) = 13 \), \( n(B) = 18 \), \( n(\overline{A} \cup \overline{B}) = 6 \), what is \( n(\overline{A} \cup \overline{B}) \)?

\[
n(\overline{A} \cup \overline{B}) = 13 + 18 - 6 = 25
\]

\[n(A) + n(B) - n(A \cap B)\]

\[
\begin{align*}
A & \cap B = 6 \\
A \cap b = 13 & \Rightarrow a = 7 \\
B \cap c = 12 & \Rightarrow c = 12 \\
n(\overline{A} \cup \overline{B}) & = a + b + c = 7 + 9 + 12 = 28
\end{align*}
\]

V. Application

Hint: When possible, start with the innermost region.

Generic 3-circles Venn:

43. \( T = \) set of tall plants; \( G = \) set of plants with green peas; \( S = \) set of plants with smooth peas.

4 had none of these characteristics \( n(T \cap G \cap S) = 4 = h \)

6 had all three characteristics \( n(T \cap G \cap S) = 6 = e \)

20 had green and smooth peas \( n(G \cap S \cap T') = 20 = e + f \rightarrow f = 20 - 6 = 14 \)

9 were tall and had green peas \( n(T \cap G \cap S') = 9 = b + e \rightarrow b = 9 - 6 = 3 \)

39 had smooth peas \( n(S) = 39 = d + e + f + g \rightarrow d + g = 39 - 20 = 19 \rightarrow g = 19 - d \)

25 had green peas \( n(G) = 25 = b + c + e + f \rightarrow c = 25 - 3 - 14 = 8 \)

22 were tall \( n(T) = 22 = a + b + d + e \rightarrow a + d = 22 - 9 = 13 \rightarrow a = 13 - d \)

50 pea plants were examined \( n(U) = 50 = a + b + c + d + e + f + g + h \)

\[
\begin{align*}
13 - d + 3 + 2 + d + 6 + 14 + 19 - d + 4 & = 50 \\
6d - d & = 50 \rightarrow d = 11 \\
a & = 13 - 11 = 2 \\
g & = 19 - 11 = 8
\end{align*}
\]

a) \( n(T \cap G) = d + e = 11 + 6 = 17 \)

b) \( n(T \cap G' \cap S') = a = 13 - 11 = 2 \)

c) \( n(T \cap G \cap S') = f = 20 - 6 = 14 \)
### 7.2 Notes O'Brien F09

**Given:**

\[ n(A \cup B) = 17 \quad n(A \cap B) = 3 \quad n(A) = 8 \]

\[ n(A' \cup B') = 21 \]

**Note:**

\[ A' \cup B' \text{ includes all elements not in } A \cap B \]

\[ \therefore \quad \text{Since } n(A \cap B') = 5 \quad \text{and } n(B \cap A') = 9 \quad \text{and} \]

\[ n(A' \cup B') = 21, \quad n((A \cup B)') = 21 - 5 - 9 = 7 \]

**De Morgan's Laws:**

\[ (A \cup B)' = A' \cap B' \]

\[ (A \cap B)' = A' \cup B' \]
7.2 # 6 p. 360

\[ A \cap B = \emptyset \]
\[ B' = a, d \]
\[ (A \cap B) \cup B' = a, b, d \]

\[ (A \cap B') \cup C \]

7.2 # 17 p. 360

\[ B' = a, d, g, h \]
\[ A = a, b, d, e \]
\[ A \cap B' = a, d \]
\[ C = d, e, f, g \]
\[ (A \cap B') \cup C = a, d, e, f, g \]

7.2 # 23 p. 360

\[ n(B) = 9 \]
\[ n(A \cap B) = 5 \]
\[ n(A \cup B) = 22 \]
\[ n(A) = n(A \cup B) + n(A \cap B) - n(B) \]
\[ n(A) = 22 + 5 - 9 = 18 \]

7.2 # 28 p. 360

\[ n(A') = 31 \]
\[ n(B) = 25 \]
\[ n(A' \cup B') = 46 \]
\[ n(A \cap B) = 12 \]
\[ a + c + d = 46 \]
\[ b = 12 \]
\[ c + d = 31 \]
\[ b + c = 25 \]
\[ 13 + d = 31 \]
\[ 12 + c = 25 \Rightarrow c = 13 \]
\[ d = 18 \]
\[ a + c + d = 46 \]
\[ a + 13 + 18 = 46 \]
\[ a = 15 \]
7.2 #32 p. 361

\( n(A) = 13 \rightarrow a+b+d+e = 13 \)

\( n(AnB\cap C) = 4 \rightarrow e = 4 \)

\( n(AnC) = 6 \rightarrow d+c = 6 \rightarrow d = 2 \)

\( n(A\cap B^c) = 6 \rightarrow a+d = 6 \rightarrow a = 4 \)

\( n(B\cap C) = 6 \rightarrow e+f = 6 \rightarrow f = 2 \)

\( n(B\cap C^c) = 11 \rightarrow b+c = 11 \)

\( n(B\cup C) = 22 \rightarrow b+c+d+e+f+g = 22 \)

\( 11 + 6 + 2 + g = 22 \)

\( g = 3 \)

\( n(A^c\cap B^c\cap C^c) = 5 \rightarrow h = 5 \)

\( n/A = \frac{a+b+d+e}{13} \)

\( 4+b+6 = 13 \rightarrow b = 3 \)

\( n(B\cap C^c) = b+c = 11 \rightarrow c = 8 \)

7.2 #36 p. 361

\( B\cap C = e+f \)

\( A = a+b+d+e \)

\( A\cup (B\cap C) = a+b+d+e+f \)

\( A\cup B = a+b+c+d+e+f \)

\( A\cup C = a+b+d+e+f+g \)

\( (A\cup B) \cap (A\cup C) = a+b+d+e+f \)
\[ n(A) = 13 \]
\[ n(A \cap B \cap C) = 4 \rightarrow e = 4 \]
\[ n(A \cup C) = 6 \]
\[ n(A \cap B') = 6 \]
\[ n(B \cap C) = 6 \]
\[ n(B \cap C') = 11 \]
\[ n(B \cup C) = 22 \]
\[ n(A' \cap B' \cap C') = 5 \rightarrow h = 5 \]

1) Since \( n(B \cap C) = 6 \), \( e + f = 6 \)
   \[ e = 4, \quad \therefore f = 2 \]

2) Since \( n(A \cup C) = 6 \), \( d + e = 6 \)
   \[ e = 4, \quad \therefore d = 2 \]

3) Since \( n(A \cap B') = 6 \), \( a + d = 6 \)
   \[ d = 2, \quad \therefore a = 4 \]

4) Since \( n(A) = 13 \), \( a + b + d + e = 13 \)
   \[ a = 4, d = 2, e = 4, \quad \therefore b = 13 - 10 = 3 \]

5) Since \( n(B \cap C') = 11 \), \( b + c = 11 \)
   \[ b = 3, \quad \therefore c = 8 \]

6) Since \( n(B \cup C) = 22 \), \( b + c + d + e + f + g = 22 \)
   \[ \rightarrow 3 + 8 + 2 + 4 + 2 + g = 22 \]
   \[ \rightarrow 19 + g = 22 \rightarrow g = 3 \]
\[ n(A) = 150 \]
\[ n(B) = 111 \]
\[ n(C) = 98 \]
\[ n(A \cap B) = 100 \]
\[ n(A \cap C) = 80 \]
\[ n(B \cap C) = 83 \]
\[ n(S \setminus (A \cap B)) = 9 \]

\[ a + 80 - x + x + 85 - x = 111 \quad \rightarrow \quad a = x + 111 - 165 \quad \rightarrow \quad a = x - 54 \]

\[ c + 80 - x + x + 83 - x = 98 \quad \rightarrow \quad c = x + 98 - 163 \quad \rightarrow \quad c = x - 65 \]

\[ g + 85 - x + x + 83 - x = 100 \quad \rightarrow \quad g = x + 100 - 148 \quad \rightarrow \quad g = x - 68 \]

\[
\begin{align*}
(x - 54) + (80 - x) + (x - 65) + (85 - x) + x + (83 - x) + (x - 68) + 9 &= 150 \\
x + 248 - 187 + 9 &= 150 \\
x + 257 - 187 &= 150 \\
x + 70 &= 150 \\
x &= 80
\end{align*}
\]

80 professors invested in stocks & bonds & CDs.
7.2 # 46 p. 363

(a) \( n(F) = 1,073,064 + 143,278 + 6041 + 18,941 = 1,246,324 \)

(b) \( n(F \cap (I \cup A)) = 6041 + 18,941 = 24,982 \)

(c) \( n(M \cup B) = 1,025,590 + 148,022 + 7106 + 21,186 + 143,278 = 1,345,242 \)

(d) \( n(W' \cup I' \cup U \bar{A}) = n(U) = 2,448,288 \)

7.2 # 61 p. 364

\( n(F \cap R \cap M) = 9 \rightarrow e = 9 \)

\( n(F' \cap R' \cap M') = 13 \rightarrow n = 13 \)

\( n(R \cap M) = 15 \rightarrow e + f = 15 \rightarrow f = 6 \)

\( n(F' \cap R) = 11 \rightarrow c + f = 11 \rightarrow c = 5 \)

\( n(R \cap M') = 17 \rightarrow b + c = 17 \rightarrow b = 12 \)

\( n(F) = 56 \rightarrow a + b + d + e = 56 \rightarrow 18 + 12 + 9 + 56 = d = 76 \)

\( n(M) = 41 \rightarrow d + e + f + g = 41 \rightarrow 17 + 9 + 6 + 56 = g = 9 \)

\( n(M') = 48 \rightarrow a + b + c + h = 48 \rightarrow a + 17 + 13 = 48 \rightarrow a = 18 \)

(a) \( n(F) = 56 \)

(b) \( n(R) = 32 \)

(c) \( n(F \cap M) = 26 \)

(d) \( n(F \cap M') = 30 \)

(e) \( n(F' \cap R') = 22 \)

(f) \( n(F \cap R) = 21 \)