7.3 Introduction to Probability

I. Random Phenomena
Random phenomena or events are those for which exact prediction is impossible. For such events, the best that we can do is to determine the probability or likelihood of each of the possible outcomes.

II. Sample Spaces
A. Experiment: an activity or occurrence with an observable result
B. Trial: a single repetition of an experiment
C. Outcomes: the possible results of each trial
D. Sample space: the set of all possible outcomes for an experiment

ex: #5 A student is asked how many points she earned on a recent 80-point test.
Sample space: \{0, 1, 2, 3, \ldots, 80\}

ex: #9 A coin is tossed, and a die is rolled
Sample space: \{H or T, 1 or 2 or 3 or 4 or 5 or 6\}
\{(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)\}

Note: If order is a consideration, some experiments may have more than one sample space (see Example 1.c. p. 365). The most useful sample spaces have equally likely outcomes, but it is not always possible to choose such a sample space.
III Events

A. Event: an event is a subset of a sample space.

\( S = \{(1,2),(1,3),(1,4),(1,5),(2,3),(2,4),(2,5),(3,4),(3,5),(4,5)\} \)

Each outcome is equally likely.

- \( E = \text{Both slips are marked with even numbers} \)
  \( E = \{(2,4)\} \) Simple Event

- \( E = \text{One number is even \& the other is odd} \)
  \( E = \{(1,2), (1,4), (2,3), (2,5), (3,4), (3,5), (4,5)\} \)

- \( E = \text{Both slips are marked with the same number} \)
  \( E = \{\} \) Impossible Event

B. Simple Event: an event with only one outcome.

C. Certain Event: If event \( E \) equals the sample space \( S \), then \( E \)
  is called a certain event \( \text{ex: rolling one die and getting a number < 8} \)

D. Impossible Event: If event \( E = \emptyset \), then \( E \) is an impossible event
  \( \text{ex: rolling one die and getting a number > 8} \)

E. Set Operations for Events

- \( E \) and \( F \) be events for a sample space \( S \).

  - \( E \cap F \) occurs when both \( E \) \& \( F \) occur.
  - \( E \cup F \) occurs when \( E \) \or \( F \) \or both occur.
  - \( E^c \) occurs when \( E \) does \text{not} occur

F. Mutually Exclusive Events: Two events are mutually exclusive if
  \( P(E \cap F) = 0 \); i.e. \( E \cap F = \emptyset \). By definition, mutually exclusive events are disjoint sets.
IV. Basic Probability Principle

Let $S$ be a sample space with $n$ equally likely outcomes, and let event $E$ be a subset of $S$ containing $m$ of these outcomes. Then the probability that event $E$ occurs, written $P(E)$, is $P(E) = \frac{n(E)}{n(S)} = \frac{m}{n}$.

The probability of an event is a number that indicates the relative likelihood of an event. For any event $E$, $0 \leq P(E) \leq 1$.

IV. A Standard Deck of Cards

A standard deck of cards has 52 cards which can be divided into four suits. Two suits are red, hearts ($\heartsuit$) and diamonds ($\diamondsuit$), and two suits are black, clubs ($\clubsuit$) and spades ($\spadesuit$). Each suit has 13 cards: Ace ($A$), King ($K$), Queen ($Q$), Jack ($J$), and cards numbered 2 to 10. The Jack, Queen, and King are called face cards.

See Figure 17 on p. 370

<table>
<thead>
<tr>
<th>Clubs</th>
<th>A♠</th>
<th>2♠</th>
<th>3♠</th>
<th>4♦</th>
<th>5♦</th>
<th>6♦</th>
<th>7♦</th>
<th>8♦</th>
<th>9♦</th>
<th>10♦</th>
<th>J♦</th>
<th>Q♦</th>
<th>K♦</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamonds</td>
<td>A♦</td>
<td>2♦</td>
<td>3♦</td>
<td>4♦</td>
<td>5♦</td>
<td>6♦</td>
<td>7♦</td>
<td>8♦</td>
<td>9♦</td>
<td>10♦</td>
<td>J♦</td>
<td>Q♦</td>
<td>K♦</td>
</tr>
<tr>
<td>Hearts</td>
<td>A♥</td>
<td>2♥</td>
<td>3♥</td>
<td>4♥</td>
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<td>6♥</td>
<td>7♥</td>
<td>8♥</td>
<td>9♥</td>
<td>10♥</td>
<td>J♥</td>
<td>Q♥</td>
<td>K♥</td>
</tr>
<tr>
<td>Spades</td>
<td>A♣</td>
<td>2♣</td>
<td>3♣</td>
<td>4♣</td>
<td>5♣</td>
<td>6♣</td>
<td>7♣</td>
<td>8♣</td>
<td>9♣</td>
<td>10♣</td>
<td>J♣</td>
<td>Q♣</td>
<td>K♣</td>
</tr>
</tbody>
</table>
A single fair die is rolled. \( S = \{1, 2, 3, 4, 5, 6\} \) \( n(S) = 6 \)

**#49** Getting a 2
\[ E = \{2\}, \quad n(E) = 1 \]
\[ p(E) = \frac{n(E)}{n(S)} = \frac{1}{6} \]

**#21** Getting a number less than 5
\[ E = \{1, 2, 3, 4\} \quad n(E) = 4 \]
\[ p(E) = \frac{4}{6} = \frac{2}{3} \]

A card is drawn from a well-shuffled deck of 52 cards. \( n(S) = 52 \)

**#25** a nine
\[ E = \{ \text{nine of clubs, nine of hearts, nine of spades, nine of diamonds} \} \]
\[ n(E) = 4 \quad \therefore \quad p(E) = \frac{4}{52} = \frac{1}{13} \]

**#31** a 2 or a queen
\[ E = \{ \text{2 of clubs, Queen of clubs, 2 of hearts, Queen of hearts, 2 of spades, Queen of spades, 2 of diamonds, Queen of diamonds} \} \]
\[ n(E) = 8 \quad \therefore \quad p(E) = \frac{8}{52} = \frac{2}{13} \]

**#43** \( E = \) worker is female; \( F = \) worker has worked less than 5 years
\( G = \) worker contributes to a voluntary retirement plan
a) \( E' = \) set of all male (non-female) workers
b) \( E \cap F = \) set of all female workers who have worked less than 5 years
c) \( E \cup G' = \) set of all workers who are either female or who do not contribute to retirement plan

Similar
to 44: Total funding = 9.0 + 1.2 + 1.0 + 2.7 + 1.1 = 15.0 billion dollars

\[ a) p(\text{Federal govt}) = \frac{9.0}{15.0} = \frac{3}{5} \]
\[ b) p(\text{Industry}) = \frac{1.0}{15.0} = \frac{1}{15} \]
\[ c) p(\text{The Institution}) = \frac{2.7}{15.0} = \frac{9}{50} \]

<table>
<thead>
<tr>
<th>Source</th>
<th>Funding for Univ Research</th>
<th>Amount (Billions)</th>
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<tr>
<td>Fed Gov't</td>
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<tr>
<td>State &amp; Local Gov</td>
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<td>Academic Inst.</td>
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<tr>
<td>Other</td>
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