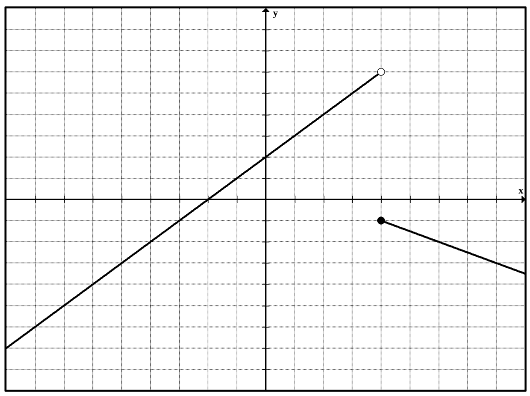
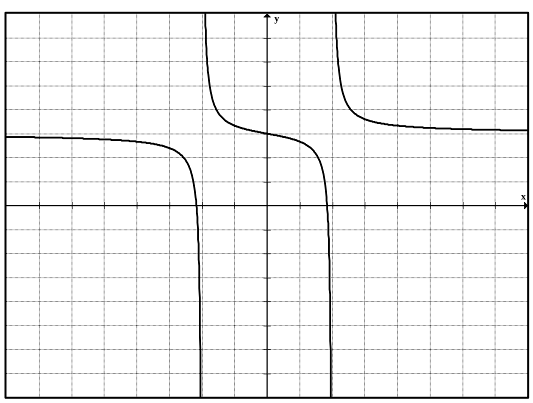
Directions: ***<u>All work should be shown and all answers should be exact and simplified (unless stated otherwise) to receive full credit on the final exam.</u>*** An acceptable graphing calculator and the departmental formula page are the only resources allowed for use during the final exam. Reference problems from the text are stated in parenthesis.

1. (1.1.11 – 13) Consider the function $f(x) = \begin{cases} x+2, \text{for } x < 4 \\ -\frac{1}{2}x+1, \text{for } x \geq 4 \end{cases}$ graphed below.



a. Find $\lim_{x\to 4^-} f(x)$. If necessary, state that the limit does not exist.
b. Find $\lim_{x\to 4^+} f(x)$. If necessary, state that the limit does not exist.
c. Find $\lim_{x\to 4} f(x)$. If necessary, state that the limit does not exist.
d. Is $f(x)$ continuous at x = 4? Why or why not?

2. (1.1.55, 61, 62) Use the graph of $g(x)$ to find each limit.



a. Find $\lim_{x\to(-2)} g(x)$. If necessary, state that the limit does not exist.
b. Find $\lim_{x\to(-\infty)} g(x)$. If necessary, state that the limit does not exist.
c. Find $\lim_{x\to\infty} g(x)$. If necessary, state that the limit does not exist.

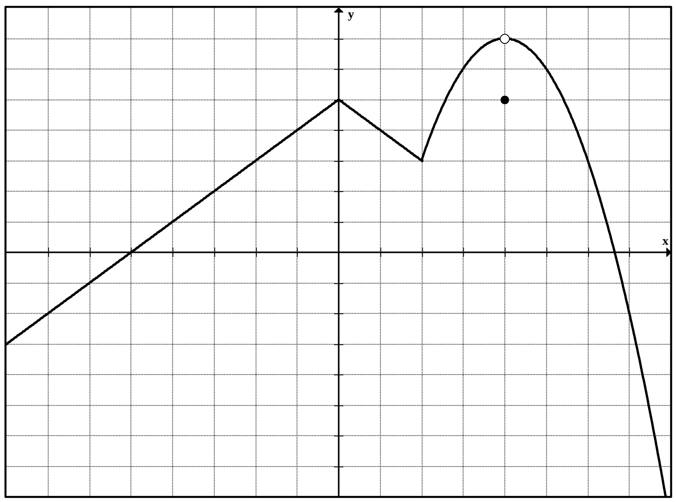
3. (1.2.13, 19, 31) Find the given limit. If necessary, state that the limit does not exist.

a. $\lim_{x\to(-3)}(-x^2+7x-8)$
b. $\lim_{x\to 9}\frac{x^2-81}{x-9}$
c. $\lim_{x\to 5}\sqrt{4x-7}$

4. (1.4.13) For $f(x) = 2x^2 - 9x + 10$, find $f'(x)$ by determining $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. Then compute $f'(1)$.

5. (1.4.21) Find an equation of the tangent line to the graph of $f(x) = -3 - 6x^2$ at $(2, -27)$.

6. (1.4.25) List the points on the graph at which the function is not differentiable.



7. (1.5.25) Find the derivative $\frac{d}{dx}\left(\sqrt[6]{x} - \frac{4}{x}\right)$

8. (1.5.31) Differentiate $y = 3x^3 - 7x^2 + 8x + 8$

9. (1.5.47) Find the derivative of $y = 3x^{-5/2} + 4x^{-1/2} + x^{12} - 8$

10. (1.5.87) The position, s, in meters of a rock in free fall near the surface of the earth is given by $s(t) = 4.69t^2$ where t is the time in seconds. Find the rate of change of the position of the rock with respect to time.

11. (1.5.107) Find the derivative of $f(x) = (5x + 1)(3x^2 + 1)$

12. (1.5.109) Find y' if $y = \frac{3x^5 - 7x^4 + 2}{x^2}$.

13. (1.6.21) Differentiate the function $y = (8x^4 - x + 7)(-x^5 + 3)$

14. (1.6.23) Differentiate the function $y = \frac{5x^2 - 2}{8x^3 + 1}$

15. (1.6.101) The cost, in dollars, of producing x jackets is given by $C(x) = 931 + 10\sqrt{x}$. Find the rate at which the average cost is changing when 324 jackets have been produced. Round to 4 decimal places as needed.

16. (1.7.7) Differentiate $y = \sqrt{x^2 + 2}$

17. (1.7.35) Differentiate $f(x) = (2x^5 - 5x^3 + 5)^{22}$

18. (1.7.75) The position of a particle moving along a line is $s = \sqrt{76 + 6t}$ with *s* in meters and *t* in seconds. Find the rate of change of the particle's position at *t* = 4 seconds.

19. (1.7.83) Suppose that the demand function for a product is given by $D(p) = \frac{55000}{p}$ and that the price *p* is a function of time given by $p = 1.6t + 12$ where *t* is in days.

    a. Find the demand as a function of time *t*.
    b. Find the rate of change of the quantity demanded when *t* = 110 days

20. (1.8.13) Find $f''(x) = 5x^2 - 12x - \frac{4}{x^3}$

21. (1.8.45) Given $s(t) = t^2 + 4t$ where *s* is in meters and *t* is in seconds, find each of the following:

a. $v(t)$
b. $a(t)$
c. The velocity and acceleration when *t* = 2 seconds

22. (1.8.55) A company determines the monthly sales, *S,* in thousands of dollars, after *t* months of marketing a product is given by $S(t) = 3t^3 - t^2 + 10t - 4$. Find $S'(4)$ and $S''(4)$.

23. (2.1.7) Find the relative extrema (if they exist) of the function $G(x) = -x^3 + 2x^2 + 4x - 2$. List the extrema as ordered pairs.

24. (2.1.23) Find the relative extrema (if they exist) of the function $F(x) = \sqrt[3]{x - 2}$. List the extrema as ordered pairs.

25. (2.1.89) The temperature of a person during an intestinal illness is given by $T(t) = -0.1t^2 + 1.2t + 98.6 \text{ for } 0 \leq t \leq 12$ where *T* is the temperature (°F) at time *t*, in days. Find the relative extrema of the function.

26. (2.2.15) For the function $f(x) = -x^3 + 6x^2 - 17$
    a. List the coordinates of any relative extrema
    b. List the coordinates of any inflection points
    c. State the intervals where the function is increasing or decreasing
    d. State the intervals where the function is concave up or concave down

27. (2.2.35) For the function $f(x) = (x + 5)^{2/7}$
    a. List the coordinates of any relative extrema
    b. List the coordinates of any inflection points
    c. State the intervals where the function is increasing or decreasing
    d. State the intervals where the function is concave up or concave down

28. (2.4.17) Find the absolute maximum and minimum values of the function over the indicated interval and indicate the x-values where they occur for $f(x) = 3 - 5x - 5x^2$; $[-2,4]$

29. (2.4.29) Find the absolute maximum and minimum values of the function over the indicated interval and indicate the x-values where they occur for $f(x) = x^4 - 18x^2 + 7; [-5,5]$

30. (2.4.97) An employee's monthly productivity *M* in number of units produced is found to be a function of the number *t* of years of service. For a certain product, a productivity function is $M(t) = -2t^2 + 80t + 150, 0 \leq t \leq 40$. In what year is the maximum productivity achieved? What is the maximum productivity?

31. (2.4.103) The total cost, *C*(x), and total revenue *R*(x) functions for producing *x* items are $C(x) = 4700 + 600x$ and $R(x) = -\frac{1}{2}x^2 + 800x$.

a. Find the total profit function *P*(x).
b. Find the number of items, *x*, for which the total profit is a maximum.

32. (2.5.13) A rectangular plot of farmland will be bounded on one side by a river and on the other 3 sides by a single-strand electric fence. With 1200 meters of wire at your disposal, what is the largest area you can enclose and what are its dimensions?

33. (2.5.15) A carpenter is building a rectangular shed with a fixed perimeter of 72 feet. What are the dimensions of the largest shed that can be built? What is its area?

34. (2.5.29) Raggs, Ltd determines that in order to sell *x* suits, the price per suit must be $p = 190 - 0.5x$. It also determines that the total cost of producing *x* suits is given by $C(x) = 2000 + 0.75x^2$

a. Find the total revenue $R(x)$
b. Find the total profit $P(x)$
c. How many suits must the company produce and sell in order to maximize profit?
d. What is the maximum profit?
e. What price per suit must be charged in order to maximize profit?

35. (2.5.31) A university is trying to determine what price to charge for tickets to football games. At a price of $16 per ticket, attendance averages 40,000 people per game. Every decrease of $2 adds 10,000 people to the average number. Every person at the game spends an average of $4.00 on concessions. What price per ticket should be charged in order to maximize revenue? How many people will attend at that price?

36. (2.5.33) An apple farm yields an average of 30 bushels of apples per tree when 16 trees are planted on an acre of ground. Each time 1 more tree is planted per acre, the yield decreases by 1 bushel per tree as a result of crowding. How many trees should be planted on an acre in order to get the highest yield?

37. (2.6.1) Let $R(x), C(x)$ and $P(x)$ be, respectively, the revenue, cost and profit, in dollars from the production and sale of *x* items. If $R(x) = 5x$ and $C(x) = 0.001x^2 + 1.2x + 60$, find each of the following:

a. $P(x)$
b. $R(100), C(100)$ and $P(100)$
c. $R'(x), C'(x)$ and $P'(x)$
d. $R'(100), C'(100)$ and $P'(100)$

38. (2.6.3) Suppose that the monthly cost, in dollars, of producing *x* chairs is $C(x) = 0.001x^3 + 0.07x^2 + 19x + 700$ and currently 25 chairs are produced monthly.

a. What is the current monthly cost?
b. What would be the additional cost of increasing production to 26 chairs monthly?
c. What is the marginal cost when $x = 25$?
d. Use marginal cost to estimate the difference in cost between producing 25 and 27 chairs per month.
e. Use the answer from part d to predict $C(27)$.

39. (2.7.3) Differentiate implicitly to find $\frac{dy}{dx}$ for $y^2 - 2x^3 = 33$ and then find the slope of the curve at the point $(2, -7)$

40. (2.7.13) Differentiate implicitly to find $\frac{dy}{dx}$ for $3x^2 + 8xy + 3y^2 + 19y - 3 = 0$ and then find the slope of the curve at the point $(-1, 0)$

41. (2.7.35) Find the rate of change of a) total revenue, b) total cost and c) total profit with respect to time when $R(x) = 3x, C(x) = 0.01x^2 + 0.5x + 10$ when $x = 25$ and $\frac{dx}{dt} = 8$ units per day.

42. (2.7.39) In a trend that some scientists attribute to global warming, a certain floating cap of sea ice has been shrinking since 1950. Average minimum size of the ice cap can be approximated by $A = \pi r^2$. In 2005, the radius of the ice cap was approximately 800 miles and was shrinking at a rate of approximately 4.2 miles per year. How fast was the area changing at that time?

43. (3.1.29) Differentiate $f(x) = x^5 - 6e^{3x}$

44. (3.1.31) Differentiate $f(x) = x^4e^{7x}$

45. (3.1.33) Differentiate $F(x) = \frac{e^{2x}}{x^4}$

46. (3.1.83) A company's total cost, in millions of dollars, is given by $C(t) = 100 - 60e^{-t}$ where *t* is the time in years since the start-up date. Find the marginal cost, $C'(t)$ and evaluate $C'(0)$ and $C'(3)$. Round your answer to the nearest thousand.

47. (3.2.55) Differentiate$f(x) = x^4 \ln 7x$.

48. (3.2.59) Differentiate $f(x) = \ln\left(\frac{x^9}{8}\right)$. Simplify your answer.

49. (3.6.1) For the demand function $D(p) = 307 - p$, find the following:
a. Find the equation for elasticity.
b. Find $E(114)$ and indicate whether the demand is elastic, inelastic or has unit elasticity.
c. Find the value(s) for *p* for which the total revenue is a maximum. Round to the nearest cent.

50. (3.6.13) A bakery works out a demand function for its chocolate chip cookies and finds it to be $D(p) = 786 - 21x$ where *D* is the demand for cookies when the price per cookie (in cents) is *p*.
   a. Find the equation for elasticity.
   b. At what price is the elasticity of demand equal to 1?
   c. At what prices is the elasticity of demand elastic?
   d. At what prices is the elasticity of demand inelastic?
   e. At a price of 12¢ per cookie, will a small increase in price cause the total revenue to increase or decrease?

51. (4.1.7) Evaluate $\int (x^2 - 9x + 2)dx$

52. (4.1.39) Evaluate $\int \left(x^4 - \frac{9}{2}\sqrt{x} + x^{-5/4}\right) dx$

53. (4.1.43) Determine the indefinite integral $\int \left(\frac{7}{x} - 2e^{5x} + \sqrt{x^5}\right) dx$

54. (4.1.65) A company finds the rate at which the quantity of a product that consumers demand changes with respect to price is given by the function $D'(x) = -\frac{4000}{x^2}$ where *x* is the price per unit in dollars. Find the demand function if it is known that 1003 units of product are demanded by the customers when the price is $4 per unit.

55. (4.1.61) A company determined that the marginal cost $C'(x)$ of producing the x-th unit of a product is given by $C'(x) = x^4 - 2x$. Find the total cost function assuming that $C(x)$ is in dollars and that fixed costs are $4000.

56. (4.2.3) Photos from Nature has found that the cost per card of producing *x* note cards is $C'(x) = -0.04x + 85$ for $x \leq 1000$ where $C'(x)$ is the cost, in cents, per card. Find the total cost of producing 650 cards.

57. (4.3.11) Find the area under the curve $y = e^{3x}$ over the interval [0, 3]. Round your answer to 3 decimal places.

58. (4.3.27) Find the area under the curve $y = x^2 + 5x + 6$ over the interval [2, 6]. Write your answer as a fraction.

59. (4.3.43) Evaluate $\int_2^5 (5x^3 + 2)dx$ and write your answer as a fraction.

60. (4.3.45) Evaluate $\int_9^{16} (\sqrt{x} - 3)dx$. Write your answer as a fraction.

61. (4.3.59) A company finds that the marginal profit, in dollars, from drilling a well that is *x* feet deep is given by $P'(x) = 2\sqrt[5]{x}$. Find the profit when a well 200 feet deep is drilled.

62. (4.3.79) A particle is released as a part of an experiment. Its speed *t* seconds after release is given by $v(t) = -0.2t^2 + 10t$ where $v(t)$ is in meters per second.
a. How far does the particle travel during the first 4 seconds?
b. How far does the particle travel in the second 4 seconds?

63. (4.3.93) Evaluate $\int_3^6 \left(\frac{4x^2-1}{2x-1}\right) dx$

64. (4.4.35) Find the area of the region bounded by the graphs of $y = x^2 + 3$ and $y = x^2$ for $x = 1, x = 2$.

65. (4.4.33) Find the area of the region bounded by the graphs of $y = 2 - x^2$ and $y = 2 - 4x$

66. (4.4.39) Find the average value of the function $f(x) = x^2 - 13$ on [0, 3].

67. (4.4.51) A company determines that its weekly online sales, $S(t)$, in hundreds of dollars, *t* weeks after online sales began can be estimated by the equation $S(t) = 7e^t$. Find the average weekly sales for the first 9 weeks after online sales began. Round your answer to the nearest cent.

68. (4.5.1) Evaluate $\int (x^3 - 6)^7 3x^2 dx$

69. (4.5.15) Evaluate $\int x^4 e^{x^5} dx$

70. (4.5.21) $\int \frac{dx}{2x+1}$

71. (5.1.1) $D(x) = -\frac{5}{6}x + 11$ is the price, in dollars per unit, that consumers are willing to pay for *x* units of an item and $S(x) = \frac{1}{2}x + 3$ is the price, in dollars per unit, that producers are willing to accept for *x* units.
a. Find the equilibrium point.
b. Find the consumer surplus at the equilibrium point.
c. Find the producer surplus at the equilibrium point.

72. (6.2.3) For the function $f(x, y) = 8x^3 + 6y^2 - 3xy$, find:
   a. f_x
   b. f_y
   c. $f_x(-2,0)$
   d. $f_y(-2,0)$

73. (6.2.9) Find f_x and f_y for $f(x, y) = -9e^{8x-3y}$

74. (6.2.31) Find the 4 second-order partial derivatives for $f(x, y) = 8x^5y^4 + 4x^7y^8$

75. (6.2.41) A sports company has the production function $p(x,y) = 2400x^{4/5}y^{1/5}$ where *p* is the number of units produced with *x* units of labor and *y* units of capital.

a. Find the number of units produced with 28 units of labor and 1087 units of capital. Round to the nearest whole number.
b. Find the marginal productivities p_x and p_y.
c. Evaluate the marginal productivities at *x* = 28 and *y* = 1087. Round to the nearest whole number.
d. Interpret the meanings of the marginal productivities from part c.

76. (6.2.39) Find the 4 second-order partial derivatives for $f(x,y) = 13y\ln x$

77. (6.3.3) Find the relative maximum and minimum values for $f(x,y) = 20xy - x^3 - 10y^2$, if they exist.

78. (6.3.7) Find the relative maximum and minimum values for $f(x,y) = x^2 + y^2 - 8x + 14y - 3$, if they exist.

79. (6.3.15) A company produces two types of solar panels per year: *x* thousand of Type A and *y* thousand of Type B. The revenue function is $R(x,y) = 4x + 3y$ in millions of dollars and the cost function is $C(x,y) = x^2 - 2xy + 7y^2 + 8x - 61y - 3$ in millions of dollars. Determine how many of each type of solar panel should be produced per year to maximize profit. Also determine the maximum profit.

80. (6.5.1) Find the maximum value of $f(x,y) = xy$ subject to $x + y = 4$.

81. (6.5.18) To fence a rectangular field, 144 feet of fence are available. Find the dimensions of the largest field that can be enclosed with this amount of fencing and find the area of this field.

82. (6.5.21) The total sales, *S*, of a company is given by the function $S(L,M) = ML - L^2$ where *M* is the cost of materials and *L* is the cost of labor. Find the maximum value of this function subject to $L + M = 100$. Round to the nearest cent.