Survey of Calculus Extra Credit Problems

Each problem, correctly worked out, with all work shown, is worth one points.
You may earn a maximum of 10 extra credit points.

1. To develop strategies to manage water quality in polluted lakes, biologists must determine the depths of sediments and the rate of sedimentation. It has been determined that the depth of sediment D(t) (in centimeters) with respect to time (in years before 1990) for Lake Coeur d'Alene, Idaho, can be estimated by the equation $D(t) = 155\left(1 - e^{-.0133\,t}\right)$.

   a. Find and interpret D(20).
   b. Find and interpret $\lim\limits_{t\to\infty} D(t)$.

2. Researchers at Iowa State University and the University of Arkansas have developed a piecewise function that can be used to estimate the body weight (in grams) of a male broiler during the first 56 days of life:

   $$W(t) = \begin{cases} 48 + 3.64t + .6363t^2 + .00963t^3 & \text{if } 1 \le t \le 28 \\ -1004 + 65.8t & \text{if } 28 < t \le 56 \end{cases}$$ where t is the age of the chicken (in days).

   a. Determine the weight of a male broiler that is 25 days old.
   b. Is W(t) a continuous function? Explain why or why not.

3. It is often difficult to evaluate the quality of products that undergo a ripening or maturation process. Researchers have successfully used ultrasonic velocity to determine the age of Mahon cheese. The age can be determined by: $M(v) = .0312443v^2 - 101.39v + 82{,}264$, $v \ge 1620$, where M(v) is the estimated age of the cheese (in days) for a velocity of v (meters per second).

   a. If Mahon cheese ripens in 150 days, determine the expected velocity of a ripe cheese.
   b. Find the instantaneous rate of change of M at v = 1700 meters per second.

4. To increase the velocity of the air flowing through the trachea when a human coughs, the body contracts the windpipe, producing a more effective cough. Philip Tuchinsky determined that the velocity of the air that is flowing through the trachea during a cough is $V = C(R_0 - R)R^2$, where C is a constant based on individual body characteristics, R_0 is the radius of the windpipe before the cough, and R is the radius of the windpipe during the cough. It can be shown that the maximum velocity of the cough occurs when V' = 0. Find the value of R that maximizes the velocity.

5. The total profit (in tens of dollars) from selling x self-help books is $P(x) = \dfrac{5x - 6}{2x + 3}$.

   a. Find the average profit if 15 books are sold.
   b. Find the marginal average profit function.

6. The amount (in grams) of a sample of Uranium 239 present after t years is given by $A(t) = 100e^{-.362\,t}$
   Find and interpret A'(3).

7. The average speed of pedestrians in a town or city is related to the population (x) by the formula $f(x) = .873 \log x - .0255$.

   a. Find and interpret f (1,000,000).
   b. Find and interpret f ' (1,000,000).

8. A study by the National Highway Traffic Safety Administration found that driver fatalities rates were highest for the youngest and oldest drivers. The rates per 1000 licensed drivers for every 100 million miles may be approximated by the function $f(x) = k(x - 49)^6 + .8$, where x is the driver's age in years and k is the constant 3.8×10^{-9}. Find and interpret the rate of change of the fatality rate when the driver is:

   a. 20 years
   b. 60 years

9. The number of people P(t) (in hundreds) infected t days after an epidemic begins is approximated by the function $P(t) = \frac{10 \ln(.19t + 1)}{.19t + 1}$. When will the number of people infected start to decline?

10. A small company manufactures and sells bicycles. The production manager has determined that the cost and demand functions for x bicycles per week are $C(x) = 10 + 5x + \frac{1}{60}x^3$ and p = 90 – x, where p is the price per bicycle and $x \geq 0$.
    a. Find the maximum weekly revenue.
    b. Find the maximum weekly profit.
    c. Find the price the company should charge to realize maximum profit.

11. The metabolic rate after a person eats a meal tends to go up and then, after some time has passed, returns to resting metabolism. This phenomenon is known as the thermic effect of food and can be described for a particular individual as $F(t) = -10.28 + 175.9\, te^{\frac{-t}{1.3}}$, where F(t) is the thermic effect of food (in kJ per hour) and t is the number of hours that have elapsed since eating a meal. Find the time after a meal when the thermic effect of food is maximized for this individual.

12. The revenue R(x) generated from sales of a certain product is related to the amount x spend on advertising by the model $R(x) = \frac{1}{15{,}000}\left(600x^2 - x^3\right)$, $0 \leq x \leq 600$, where x and R(x) are in thousands of dollars. Is there a point of diminishing returns for this function? If so, where is it and what does it mean?

13. A company wants to manufacture cylindrical aluminum cans with a volume of 1000 cm^3 (1 liter). What should the radius and height of the can be to minimize the amount of aluminum used?

14. In 1968, Terrence Wales described the demand for distilled spirits as q = f (p) = –.00375p + 7.87, where p represents the retail price of a case of liquor in dollars per case. q represents the average number of cases purchased per year by a consumer. Find and interpret the elasticity of demand when p = $118.30 per case.

15. A cone-shaped icicle is dripping from the roof. The radius of the icicle is decreasing at a rate of .2 cm per hour, while the length is increasing at a rate of .8 cm per hour. If the icicle is currently 4 cm in radius and 20 cm long, is its volume increasing or decreasing, and at what rate?