Survey of Calculus Exam 1 Review: 1.1 – 1.8

Be sure you have memorized and know how to use the definition of the derivative and the differentiation formulas.

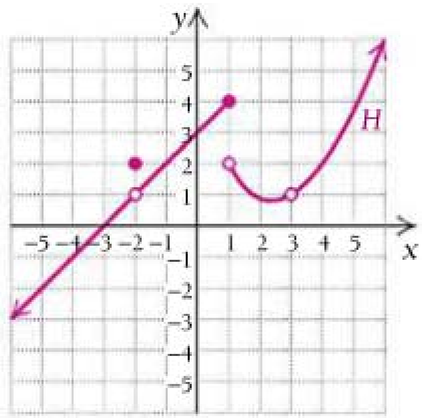
Show all of your support work on all problems. The more work you show me on the exam, the more partial credit I can give you. Try working these problems with your book, homework, and notes closed. That is the best way to prepare for the actual exam.

1.1 Limits: A Numerical and Graphical Approach

1. Given $f(x)=\begin{cases} 6-x, & \text{if } x<2 \\ 7x-10, & \text{if } x\geq 2 \end{cases}$, use the numerical (table) method to find the following limits.

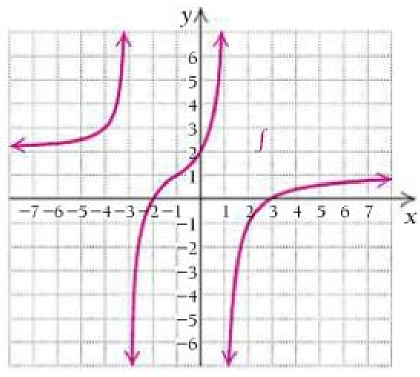
   a. $\lim_{x\to 2^-} f(x)$ b. $\lim_{x\to 2^+} f(x)$ c. $\lim_{x\to 2} f(x)$

2. Use the following graph of H to find each limit. When necessary, state that the limit does not exist.



   a. $\lim_{x\to 1^-} f(x)$ b. $\lim_{x\to 1^+} f(x)$ c. $\lim_{x\to 1} f(x)$

3. Use the following graph of f to find each limit. When necessary, state that the limit does not exist.



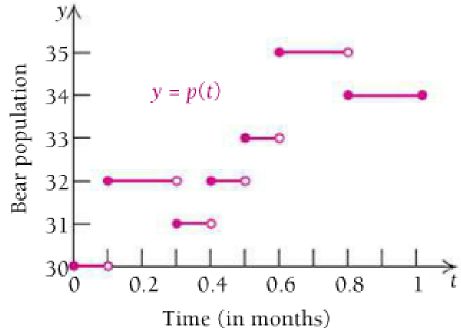
   a. $\lim_{x\to -3} f(x)$ b. $\lim_{x\to -1} f(x)$ c. $\lim_{x\to -\infty} f(x)$ d. $\lim_{x\to \infty} f(x)$

4. Graph the following function and then find the specified limits. When necessary, state that the limit does not exist. Include an accurate graph with your answers.

$$g(x) = \frac{1}{x+2} + 4$$

a. $\lim_{x \to \infty} f(x)$ b. $\lim_{x \to -1} f(x)$ c. $\lim_{x \to -2} f(x)$

5. The population of bears in a certain region is given in the graph of p below. Time, t, is measured in months. Use the graph to find the following limits.



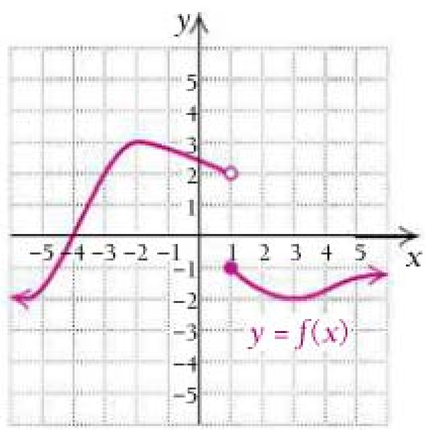
a. $\lim_{t \to .7^-} p(t)$ b. $\lim_{t \to .7^+} p(t)$ c. $\lim_{t \to .7} p(t)$

1.2 Algebraic Limits and Continuity

6. Find the following limits. When necessary, state that the limit does not exist.

a. $\lim_{x \to 3} \frac{x^2 - x - 1}{\sqrt{x+1}}$ b. $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$ c. $\lim_{x \to 3} \sqrt{x^2 - 16}$

7. Use the graph below to answer the following questions. If an expression does not exist, state that fact.



a. Find $\lim_{x \to 1^-} f(x)$, $\lim_{x \to 1^+} f(x)$, $\lim_{x \to 1} f(x)$.

b. Find f(1).

c. Is f continuous at x = 1? Why or why not?

d. Find $\lim_{x \to -2} f(x)$.

e. Find f(–2).

f. Is f continuous at x = –2? Why or why not?

8. Is the function $G(x)=\begin{cases}\frac{1}{2}x+1, & \text{for } x<4\\ -x+5, & \text{for } x>4\end{cases}$ continuous at x = 4? Why or why not? Give all reasons.

1.3 Average Rates of Change

9. Given the function $f(x)=x^2+4x-3$,

a. Find a simplified form of the difference quotient, and then

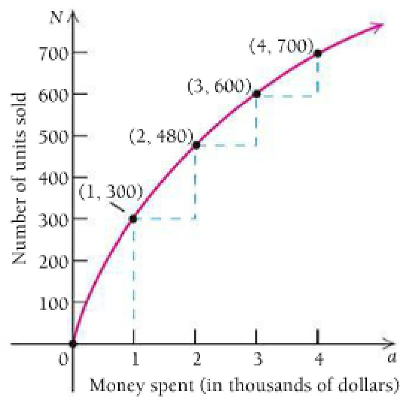
b. complete the adjacent table.

x	h	$\frac{f(x+h)-f(x)}{h}$
5	2	
5	1	
5	.1	
5	.01	

10. The following graph shows a typical response to advertising.
After an amount *a* is spent on advertising, the company sells *N(a)* units of a product.

a. Find the average rate of change of *N* as *a* changes from 0 to 1; from 1 to 2; from 2 to 3; and from 3 to 4.

b. Why do you think the average rates of change are decreasing as *a* increases?



11. The amount of money, A(t), in a savings account that pays 6% interest, compounded quarterly for t years, when an initial investment of $2,000 is made, is given by

$A(t)=2000(1.015)^{4t}$.

Find $\frac{A(5)-A(3)}{5-3}$ and interpret this result.

12. At the beginning of a trip, the odometer on a car reads 30,680, and the car has a full tank of gas. At the end of the trip, the odometer reads 31,077. It takes 13.5 gal of gas to refill the tank.

a. What is the average rate at which the car was traveling, in miles per gallon?

b. What is the average rate of gas consumption in gallons per mile?

1.4 Differentiation Using Limits of Difference Quotients

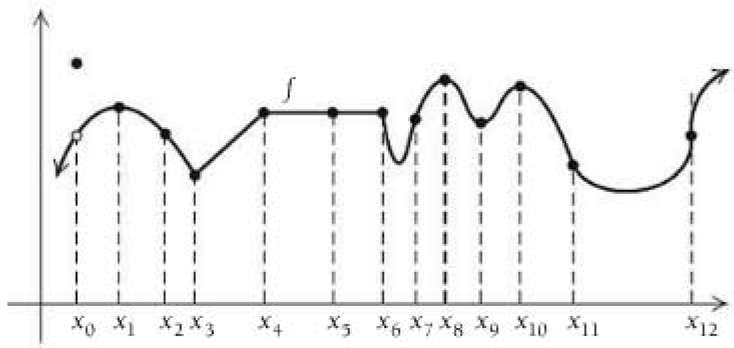
13. For the given functions, find $f'(x)$ by determining $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$.

a. $f(x)=2x+3$ b. $f(x)=3x^2-4x+2$ c. $f(x)=\frac{2}{x}$

14. For the given functions, find $f'(x)$ by determining $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. Then write the equation of the tangent line to the graph of the function at the given point.

a. $f(x)=x^3$; (4, 64) b. $f(x)=x^2-2x$; (–2, 8)

15. List the points on the graph at which the function is not differentiable and state why it is not differentiable at each point.



<u>1.5 Differentiation Techniques: The Power and Sum – Difference Rules</u>

16. Find the derivative for each function.

a. $f(x)=4x^3-7x^2+8x+17$ b. $y=3x^{-\frac{2}{3}}+x^{\frac{3}{4}}+x^{\frac{6}{5}}+\frac{8}{x^3}$

17. If $y=\sqrt[3]{x}+\sqrt{x}$, find $\left.\frac{dy}{dx}\right|_{x=64}$.

18. Use calculus (not a graphing calculator) to find the equation of the tangent line to the graph of $f(x)=x^2-\sqrt{x}$ at (1, 0).

19. Given $g(x)=-x^3+6x^2$, find the points on its graph at which the tangent line is horizontal. If none exist, state that fact.

20. The median weight of a boy whose age is between 0 and 36 months can be approximated by the function
$w(t)=.000758t^3-.0596t^2+1.82t+8.15$
where *t* is measured in months and *w* is measured in pounds.

Use this function to find the following:

a. the rate of change of weight with respect to time.

b. the weight of the baby at age 10 months.

c. the rate of change of the baby's weight with respect to time at age 10 months.

1.6 Differentiation Techniques: The Product and Quotient Rules

21. Differentiate two ways: first, by using the Product Rule; then by multiplying the expressions before differentiating.

$$h(x) = (2x+5)(3x^2 - 4x + 1)$$

22. Differentiate two ways: first by using the Quotient Rule; then by dividing the expressions before differentiating.

$$y = \frac{t^2 - 16}{t+4}$$

23. Differentiate each function.

a. $y = \dfrac{3x^4 + 2x}{x^3 - 1}$ b. $F(x) = (-3x^2 + 4x)(7\sqrt{x} + 1)$

24. Find an equation of the tangent line to the graph of $y = \dfrac{\sqrt{x}}{x+1}$ at x = 1.

25. Sparkle Pottery has determined that the cost, in dollars, of producing *x* vases is given by $C(x) = 4300 + 2.1x^{.6}$. If the revenue from the sale of *x* vases is given by $R(x) = 65x^{.9}$, find the rate at which the average profit per vase is changing when 50 vases have been made and sold.

1.7 The Chain Rule

26. Differentiate each function.

a. $y = \sqrt{4x^2 + 1}$ b. $f(x) = (x-4)^8 (2x+3)^6$ c. $G(x) = \left(\dfrac{3x-1}{5x+2}\right)^4$

27. Given $y = u(u+1)$ and $u = x^3 - 2x$, find $\dfrac{dy}{du}$, $\dfrac{du}{dx}$, and $\dfrac{dy}{dx}$.

28. A total-cost function is given by $C(x) = 2000(x^2 + 2)^{\frac{1}{3}} + 700$, where C(x) is the total cost, in thousands of dollars, of producing *x* items. Find the rate at which the total cost is changing when 20 items have been produced.

29. A company is selling laptop computers. It determines that its total profit, in dollars, is given by $P(x) = .08x^2 + 80x$, where *x* is the number of units produced and sold. Suppose that *x* is a function of time, in months, where $x = 5t + 1$.

a. Find the total profit as a function of time *t*.

b. Find the rate of change of total profit when t = 48 months.

1.8 Higher-Order Derivatives

30. Find the second derivative of the following functions.

a. $y = 5x^3 + 4x$ b. $f(x) = x^3 - \frac{5}{x}$ c. $g(x) = (2x^2 - 3x + 1)^{10}$

d. $h(x) = (x^2 + 3)(4x - 1)$ e. $F(x) = \frac{2x + 3}{5x - 1}$

31. For $y = x^7 - 8x^2 + 2$, find $\frac{d^6y}{dx^6}$.

32. Given $s(t) = t^2 - \frac{1}{2}t + 3$, where *s* is in meters and *t* is in seconds, find each of the following.

a. $v(t)$

b. $a(t)$

c. The velocity and acceleration when t = 1 sec.

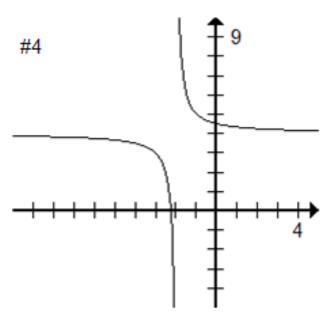
33. A company determines that monthly sales *S*, in thousands of dollars, after *t* months of marketing a product is given by $S(t) = 2t^3 - 40t^2 + 220t + 160$.

a. Find $S'(1)$, $S'(2)$, and $S'(4)$.

b. Find $S''(1)$, $S''(2)$, and $S''(4)$.

Answers

1. a. 4 b. 4 c. 4
2. a. 4 b. 2 c. DNE
3. a. DNE b. 1 c. 2 d. 1
4. a. 4 b. 5 c. DNE
5. a. 35 b. 35 c. 35
6. a. 5/2 or 2.5 b. 5 c. DNE
7. a. –1, 2, DNE b. –1 c. No, the limit does not exist.
   d. 3 e. 3 f. Yes, $\lim_{x \to -2} f(x) = f(-2)$.



8. No, $\lim_{x \to 4} G(x)$ does not exist and G(4) does not exist.
9. a. 2x + h + 4 b. 16, 15, 14.1, 14.01
10. a. 300 units/thousands of dollars; 180 units/thousands of dollars; 120 units/thousands of dollars; 100 units/thousands of dollars
    b. As more and more people see the ads, they become less effective and the number of units sold levels off.
11. $151.24 is the average annual increase in the account value from the 3rd to the 5th year.
12. a. 29.4 mi/gal b. .034 gal/mi

13. a. $f'(x)=2$ b. $f'(x)=6x-4$ c. $f'(x)=-\frac{2}{x^2}$

14. a. $f'(x)=3x^2$; y = 48x – 128 b. $f'(x)=2x-2$; y = –6x – 4

15. x_0 – discontinuity; x_3 – corner; x_4 – corner; x_6 – corner; x_{12} – vertical tangent line

16. a. $f'(x)=12x^2-14x+8$ b. $y'=-2x^{-\frac{5}{3}}+\frac{3}{4}x^{-\frac{1}{4}}+\frac{6}{5}x^{\frac{1}{5}}-\frac{24}{x^4}$

17. $\left.\frac{dy}{dx}\right|_{x=64}=\frac{1}{12}$

18. $y=\frac{3}{2}x-\frac{3}{2}$

19. (0, 0) and (4, 32)

20. a. $w'(t)=.002274t^2-.1192t+1.82$ b. ~21 lb c. ~.86 lb per month

21. $h'(x)=18x^2+14x-18$

22. $y'=1$, for $t\neq-4$

23. a. $y'=\frac{3x^6-16x^3-2}{\left(x^3-1\right)^2}$ b. $F'(x)=-\frac{105}{2}x^{\frac{3}{2}}-6x+42x^{\frac{1}{2}}+4$

24. $y=\frac{1}{2}$

25. $1.64 per vase

26. a. $y'=\frac{4x}{\sqrt{4x^2+1}}$ b. $f'(x)=4(x-4)^7(2x+3)^5(7x-6)$ c. $G'(x)=\frac{44(3x-1)^3}{(5x+2)^5}$

27. $\frac{dy}{du}=2u+1$, $\frac{du}{dx}=3x^2-2$, $\frac{dy}{dx}=\left(2x^3-4x+1\right)\left(3x^2-2\right)$

28. $489,574 per item

29. a. $P(t)=2t^2+400.8t+80.08$ b. $592.80 per month

30. a. $y''=30x$ b. $f''(x)=6x-\frac{10}{x^3}$ c. $g''(x)=10\left(2x^2-3x+1\right)^8\left(152x^2-228x+85\right)$

d. $h''(x)=24x-2$ e. $F''(x)=\frac{170}{(5x-3)^3}$

31. $\frac{d^6y}{dx^6}=5040x$

32. a. $v(t)=2t-\frac{1}{2}$ b. $a(t)=2$ c. $v(1)=1.5$ m per sec ; $a(1)=2$ m per sec^2

33. a. $S'(1)=\$146{,}000$ per month , $S'(2)=\$84{,}000$ per month , and $S'(4)=-\$4000$ per month .

b. $S''(1)=-\$68{,}000$ per month^2 , $S''(2)=-\$56{,}000$ per month^2 , and $S''(4)=-\$32{,}000$ per month^2 .