Survey of Calculus Exam 2 Review: 2.1 – 3.6

Rework Exam 1 to help you prepare for the part of the second exam which covers 1.1 – 1.8.

Solve all the problems using calculus, not a graphing calculator or algebra alone. Show all of your support work on all problems. The more work you show me on the exam, the more partial credit I can give you. Try working these problems with your book, homework, and notes closed. That is the best way to prepare for the actual exam.

2.1 Using First Derivatives to Find Maximum and Minimum Values and Sketch Graphs

1. Find the relative extrema of each function, if they exist. List each extremum along with the x-value at which it exists. Then sketch a graph of the function. **[Use calculus, not a graphing calculator to find the extrema.]**

a. $f(x) = 12 + 9x - 3x^2 - x^3$

b. $g(x) = (x+3)^{\frac{2}{3}} - 5$

c. $h(x) = \dfrac{4x}{x^2 + 1}$

2. The temperature of a person during an intestinal illness is given by $T(t) = -.1t^2 + 1.2t + 98.6,\ 0 \le t \le 12$, where T is the temperature $(^\circ F)$ at time t, in days. Find the relative extrema and sketch the graph of the function. **[Use calculus, not a graphing calculator to find the extrema.]**

2.2 Using Second Derivatives to Find Maximum and Minimum Values and Sketch Graphs

3. Find all extrema of the given function and classify each as a maximum or minimum. Use the Second Derivative Test if possible. **[Do not use a graphing calculator to find the extrema.]**

$f(x) = 8x^3 - 6x + 1$

4. Sketch the graph of each function. List the coordinates of where extrema or points of inflection occur. State where the function is increasing or decreasing, as well as where it is concave up or concave down. **[Use calculus, not a graphing calculator to find the extrema and inflection points.]**

a. $g(x) = x^3 - 6x^2 + 9x + 1$

b. $h(x) = -2(x-4)^{\frac{2}{3}} + 5$

c. $j(x) = \dfrac{8x}{x^2 + 1}$

5. The percentage of the U.S. civilian labor force age 45 – 54 may be modeled by the function $f(x) = .025x^2 - .71x + 20.44$ where x is the number of years after 1970. Sketch the graph of this function for $0 \le x \le 30$. **[Use calculus, not a graphing calculator to find the extrema and inflection points.]**

2.4 Using Derivatives to Find Absolute Maximum and Minimum Values

6. Use calculus to find the absolute maximum and minimum values of each function, if they exist, over the indicated interval. Also indicate the x-values at which each extremum occurs. When no interval is specified, use the real line, $(-\infty, \infty)$.

a. $f(x) = x^3 - x^2 - x + 2$; $[-1, 2]$

b. $g(x) = 12x - x^2$

c. $h(x) = x^2 + \dfrac{432}{x}$; $(0, \infty)$

7. An employee's monthly production M, in number of units produced, is found to be a function of t, the number of years of service. For a certain product, a productivity function is given by $M(t) = -2t^2 + 100t + 180$, $0 \le t \le 40$. Use calculus to find the maximum productivity and the year in which it is achieved.

2.5 Maximum-Minimum Problems; Business and Economics Applications

Solve the following application problems using calculus, not a graphing calculator or algebra alone.

8. Of all rectangles that have a perimeter of 60 ft, find the dimensions of the one with the largest area. What is its area?

9. From a thin piece of cardboard 8 in. by 8 in., square corners are cut out so that the sides can be folded up to make a box. What dimensions will yield a box of maximum volume? What is the maximum volume?

10. A company is constructing an open-top, square-based, rectangular metal tank that will have a volume of 32 ft^3. What dimensions will minimize surface area? What is the minimum surface area?

11. In a 300-unit hotel, all rooms are occupied when the hotel charges $80 per day for a room. For every increase of x dollars in the daily room rate, there are x rooms vacant. Each occupied room costs $22 per day to service and maintain. What should the hotel charge per day in order to maximize profit?

2.6 Marginals and Differentials

12. Given $C(x) = 62x^2 + 27{,}500$ and $R(x) = x^3 - 12x^2 + 40x + 10$, find each of the following.

a. Total profit, P(x)

b. Total cost, revenue, and profit from the production and sale of 50 units of the product

c. The marginal cost, revenue, and profit when 50 units are produced and sold

13. Suppose the daily cost, in dollars, of producing x radios is $C(x) = .002x^3 + .1x^2 + 42x + 300$ and currently 40 radios are produced daily.

a. What is the current daily cost?

b. What would be the additional daily cost of increasing production to 41 radios daily?

c. What is the marginal cost when x = 40?

d. Use the marginal cost to estimate the daily cost of increasing production to 42 radios daily.

2.7 Implicit Differentiation and Related Rates

14. Differentiate implicitly to find $\frac{dy}{dx}$. Then find the slope of the curve at the given point.

a. $4x^3 - y^4 - 3y + 5x + 1 = 0$; (1, –2)

b. $x^3 - x^2y^2 = -9$; (3, –2)

15. Differentiate $x^2 + 2xy = 3y^2$ implicitly to find $\frac{dy}{dx}$.

16. Differentiate the given demand function implicitly to find $\frac{dp}{dx}$.

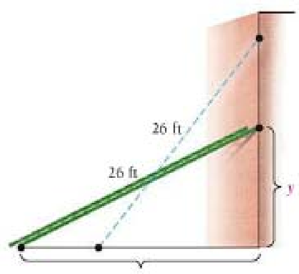
$$\frac{xp}{x+p} = 2$$

17. Find the rates of change of total revenue, cost, and profit with respect to time. Assume that R(x) and C(x) are in dollars.

$R(x) = 50x - .5x^2$ and $C(x) = 4x + 10$ when x = 30 and $\frac{dx}{dt} = 20$ units per day.

18. The area of a healing wound is given by $A = \pi \cdot r^2$. The radius is decreasing at the rate of 1 millimeter per day (–1 mm/day) at the moment when r = 25 mm. How fast is the area decreasing at that moment?

19. A ladder 26 ft long leans against a vertical wall. If the lower end is being moved away from the wall at the rate of 5 ft/s, how fast is the height of the top changing (this will be a negative rate) when the low end is 10 ft from the wall.



20. The volume of a cantaloupe is given by $V = \frac{4}{3}\pi \cdot r^3$. The radius is growing at the rate of .7 cm/week, at a time when the radius is 7.5 cm. How fast is the volume changing at that moment?

3.1 Exponential Functions

21. Graph $f(x)=\left(\frac{2}{3}\right)^x$.

22. Differentiate.

a. $G(x)=x^3-5e^{2x}$

b. $g(x)=\frac{e^{3x}}{x^6}$

c. $y=xe^{-2x}+e^{-x}+x^3$

23. Differentiate.

a. $y=e^{\sqrt{x-7}}$

b. $f(x)=\left(5x^2-8x\right)e^{x^2-4x}$

24. Graph $F(x)=-e^{\frac{1}{3}x}$. Then find critical values, inflection points, intervals over which the function is increasing or decreasing, and the concavity.

25. More Americans are buying organic fruit and vegetables and products made with organic ingredients. The amount A(t), in billions of dollars, spent on organic food and beverages t years after 1995 can be approximated by $A(t)=2.43e^{.18t}$.

a. Estimate the amount Americans spent on organic food and beverages in 2009.

b. Estimate the rate at which spending on organic food and beverages was growing in 2006.

3.2 Logarithmic Functions

26a. Write an equivalent exponential equation for $\log_{27} 3=\frac{1}{3}$.

b. Write an equivalent logarithmic equation for $Q^n=T$.

27a. Given $\log_b 3=1.099$ and $\log_b 5=1.609$, find $\log_b \frac{5}{3}$.

b. Given $\ln 4=1.3863$ and $\ln 5=1.6094$, find ln 80. Do not use the LN key on your calculator.

28. Differentiate.

a. $y=x^6\ln x-\frac{1}{4}x^4$

b. $g(x)=x^5\ln(3x)$

c. $h(x)=\frac{\ln x}{x^5}$

29. Differentiate.

a. $y=\ln\left(3x^2+2x-1\right)$

b. $g(x)=e^{2x}\ln x$

c. $h(x)=(\ln x)^4$

30. A model for consumers' response to advertising is given by $N(a)=1000+200\ln a$, $a \geq 1$, where N(a) is the number of units sold and a is the amount spent on advertising, in thousands of dollars.

a. How many units were sold after spending $1000 on advertising?

b. Find $N'(a)$ and $N'(10)$.

c. Find the maximum and minimum values of N, if they exist.

3.6 An Economics Application: Elasticity of Demand

31. For the given demand functions, find the following:

1. The elasticity
2. The elasticity at the given price, stating whether the demand is elastic or inelastic
3. The value(s) of x for which total revenue is a maximum (assume that x is in dollars).

a. $q=D(x)=\sqrt{300-x}$; x = 250

b. $q=D(x)=200e^{-.05x}$; x = 80

c. $q=D(x)=\dfrac{100}{(x+3)^2}$; x = 1

32. Good Times Bakers works out a demand function for its chocolate chip cookies and finds it to be q = D(x) = 967 – 25x, where q is the quantity of cookies sold when the price per cookie, in cents, is x.

a. Find the elasticity.

b. At what price is the elasticity of demand equal to 1?

c. At what price is the elasticity of demand elastic?

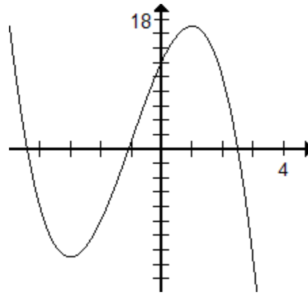
d. At what price is the elasticity of demand inelastic?

e. At what price is the revenue a maximum?

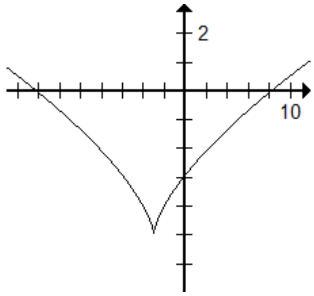
f. At a price of 20¢ per cookie, will a small increase in price cause the total revenue to increase or decrease?

Answers

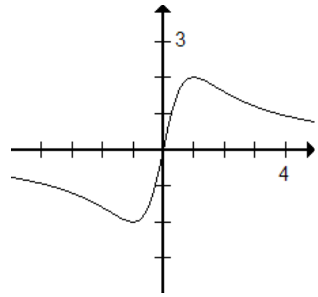
1a. Relative minimum at (–3, –15); relative maximum at (1, 17)



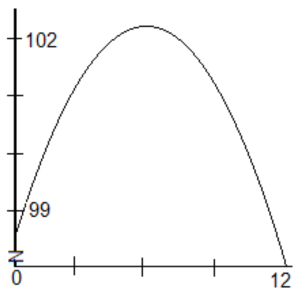
1b. Relative minimum at (–3. –5)



1c. Relative minimum at (–1, –2); relative maximum at (1, 2)

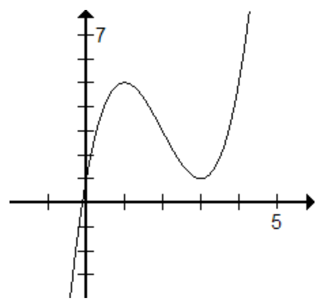


2. Relative maximum at (6, 102.2)

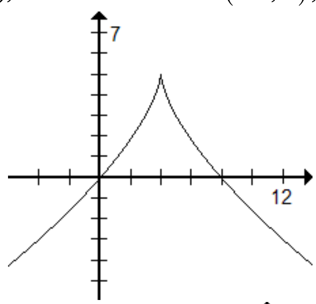


3. Relative minimum is $f\left(\frac{1}{2}\right) = -1$; relative maximum is $f\left(-\frac{1}{2}\right) = 3$

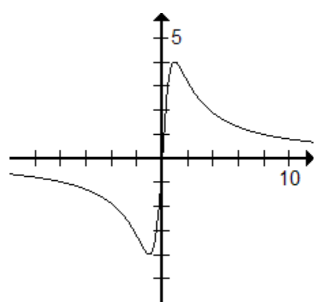
4a. Relative minimum at (3, 1); relative maximum at (1, 5); inflection point at (2, 3); increasing on $(-\infty, 1)$ & $(3, \infty)$; decreasing on (1, 3); concave down on $(-\infty, 2)$; concave up on $(2, \infty)$



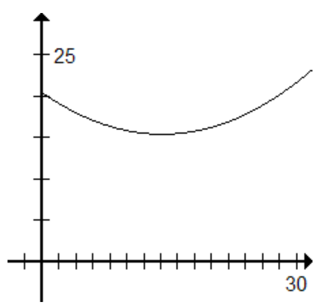
4b. Relative maximum at (4, 5); no inflection points; increasing on $(-\infty, 4)$; decreasing on $(4, \infty)$; concave up on $(-\infty, 4)$ and $(4, \infty)$



4c. Relative minimum at (–1, –4); relative maximum at (1, 4); inflection points at $\left(-\sqrt{3}, -2\sqrt{3}\right)$, (0, 0), $\left(\sqrt{3}, 2\sqrt{3}\right)$; increasing on (–1, 1); decreasing on $(-\infty, -1)$ and $(1, \infty)$; concave up on $\left(-\sqrt{3}, 0\right)$ & $\left(\sqrt{3}, \infty\right)$; concave down on $\left(-\infty, -\sqrt{3}\right)$ & $\left(0, \sqrt{3}\right)$



5. Relative minimum at (14.2, 15.399);
No inflection points;
decreasing on (0, 14.2)
increasing on (14.2, 30)
concave up on (0, 30)



6a. Absolute maximum: 4 at x = 2; absolute minimum: 1 at x = –1 and x = 1.
b. Absolute maximum: 36 at x = 6
c. Absolute minimum: 108 at x = 6

7. 1430 units; 25 years of service

8. Maximum area = 225 ft^2; width = 15 ft; length = 15 ft

9. Width: $\frac{16}{3}$ in.; length: $\frac{16}{3}$ in.; height: $\frac{4}{3}$ in.; maximum volume: $\frac{1024}{27}$ in^3

10. Dimensions: 4 ft by 4 ft by 2 ft; minimum surface area = 48 ft^2

11. $201 per room

12a. $P(x) = x^3 - 74x^2 + 40x - 27{,}490$
b. C(50) = $182, 500; R(50) = $97,010; P(50) = –$85,490
c. $C'(50) = \$6200$; $R'(50) = \$6340$; $P'(50) = \$140$

13a. $2268 b. $59.94 c. $59.60 d. $2387.20

14a. $\frac{dy}{dx} = \frac{12x^2 + 5}{4y^3 + 3}$; $-\frac{17}{29}$
b. $\frac{dy}{dx} = \frac{3x - 2y^2}{2xy}$; $-\frac{1}{12}$

15. $\frac{dy}{dx} = \frac{y + x}{3y - x}$

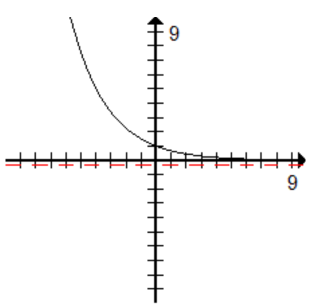
16. $\frac{dp}{dx} = \frac{2 - p}{x - 2}$

17. $\frac{dR}{dt} = \$400/\text{day}$; $\frac{dC}{dt} = \$80/\text{day}$; $\frac{dP}{dt} = \$320/\text{day}$

18. Decreasing by 157.08 mm^2/day

19. $-2\frac{1}{12}$ ft/sec

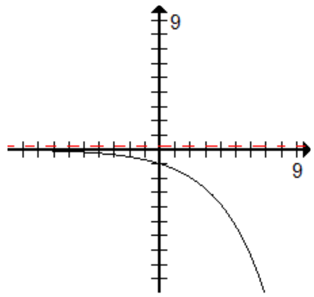
20. 494.8 cm^3/week

21.


22. a. $G'(x) = 3x^2 - 10e^{2x}$ b. $g'(x) = \dfrac{3e^{3x}(x-2)}{x^7}$ c. $y' = -2xe^{-2x} + e^{-2x} - e^{-x} + 3x^2$

23. a. $y' = \dfrac{e^{\sqrt{x-7}}}{2\sqrt{x-7}}$ b. $f'(x) = (5x^2 - 8x)e^{x^2-4x}(2x-4) + (10x-8)e^{x^2-4x}$

24. No critical values
No inflection points
Decreasing on $(-\infty, \infty)$
Concave down on $(-\infty, \infty)$



25. a. \$30.20 billion b. \$3.17 billion/year

26. a. $27^{\frac{1}{3}} = 3$ b. $\log_Q T = n$

27. a. .51 b. 4.382

28. a. $y' = x^5 + 6(\ln x)x^5 - x^3$ b. $g'(x) = x^4 + 5x^4 \ln(3x)$ c. $h'(x) = \dfrac{1 - 5\ln x}{x^6}$

29. a. $y' = \dfrac{2(3x+1)}{3x^2 + 2x - 1}$ b. $g'(x) = \dfrac{e^{2x}}{x} + 2e^{2x}\ln x$ c. $h'(x) = \dfrac{4(\ln x)^3}{x}$

30. a. 1000 units b. $N'(a) = \dfrac{200}{a}$, $N'(10) = 20$ units per \$1000 spent on advertising
c. minimum is 1000 units

31a. 1. $E(x) = \dfrac{x}{2(300-x)}$ 2. 2.5, elastic 3. \$200
b. 1. E(x) = .05x 2. 4, elastic 3. \$20
c. 1. $E(x) = \dfrac{2x}{x+3}$ 2. 0.5, inelastic 3. \$3

32. a. $E(x) = \dfrac{25x}{967 - 25x}$ b. approximately 19¢ c. prices greater than 19¢
d. prices less than 19¢ e. about 19¢ f. decrease