2.6   Marginals and Differentials

I.   Marginal Cost, Revenue, and Profit

   A.   Marginal Cost

   If C(x) is the cost of producing x units and C(x + 1) is the cost of producing x + 1 units,
   then C(x + 1) – C(x) is the additional cost of producing one more unit (after x units have been
   produced). We call this the <u>marginal cost</u>. It is equal to the derivative of the cost function.
   In symbols,

   $C'(x) \approx C(x+1) - C(x)$.

   The cost to produce the (x + 1)st item can be approximated by   C(x + 1) ≈ C(x) + C'(x).

   B.   Marginal Revenue

   If R(x) is the revenue from selling x units and R(x + 1) is the revenue from selling x + 1 units,
   then R(x + 1) – R(x) is the additional revenue from selling one more unit (after x units have been
   sold). We call this the <u>marginal revenue</u>. It is equal to the derivative of the revenue function.
   In symbols,

   $R'(x) \approx R(x+1) - R(x)$

   The revenue from selling the (x + 1)st item can be approximated by   R(x + 1) ≈ R(x) + R'(x).

   C.   Marginal Profit

   If P(x) is the profit from producing and selling x units and P(x + 1) is the profit from producing and
   selling x + 1 units, then P(x + 1) – P(x) is the additional profit from producing and selling one more
   unit (after x units have been produced and sold). We call this the <u>marginal profit</u>. It is equal to
   the derivative of the profit function. In symbols,

   P'(x) ≈ P(x + 1) – P(x)

   The profit from producing and selling the (x + 1)st item can be approximated by
   P(x + 1) ≈ P(x) + P'(x).

   D.   The Relationships between Cost, Revenue, and Profit and Marginal Cost, Revenue, and Profit

   Profit = Revenue – Cost   or   P(x) = R(x) – C(x).

   Marginal Profit = Marginal Revenue – Marginal Cost   or   P'(x) = R'(x) – C'(x)

II.   Examples

   Example 1: Ex 1 pg. 278

   Given   C(x) = 62x² + 27,500   and   R(x) = x³ – 12x² + 40x + 10,

   a.   Find the Total Profit function, P(x).

   P(x) = R(x) – C(x)

   P(x) = x³ – 12x² + 40x + 10 – (62x² + 27,500) = x³ – 12x² + 40x + 10 – 62x² – 27,500

   P(x) = x³ – 74x² + 40x – 27,490

   b.   Find the total cost, revenue, and profit from the production and sale of 50 units of the product.

   C(50) = 62(50)² + 27,500 = $182,500
   The total cost of producing the first 50 units is $182,500.

   R(50) = (50)³ – 12(50)² +40(50) + 10 = $97,010
   The total revenue from selling the first 50 units is $97,010.

P(50) = R(50) – C(50) = $97,010 – $182,500 = –$85,490
The total loss from producing and selling the first 50 units is $85,490.

c.   Find the marginal cost, revenue, and profit when 50 units are produced and sold.

C'(x) = 124x   therefore   C'(50) = 124(50) = $6200
Once 50 units have been produced, the approximate cost of the 51st unit (marginal cost) is $6200.

R'(x) = 3x^2 – 24x + 40   therefore   R'(50) = 3(50)^2 – 24(50) + 40 = $6340
Once 50 units have been sold, the approximate revenue from the 51st unit (marginal revenue) is $6340.

P'(50) = R'(50) – C'(50) = $6340 – $6200 = $140
Once 50 units have been produced and sold, the approximate profit from the 51st unit (marginal profit) is $140.

Example 2: #3 pg. 285

Suppose that the monthly cost, in dollars, of producing x chairs is   C(x) = .001x^3 + .07x^2 + 19x + 700, and currently 25 chairs are produced monthly.

a.   What is the current monthly cost?

C(25) = .001(25)^3 + .07(25)^2 + 19(25) + 700 = $1,234.375 ≈ $1,234.38

b.   What would be the additional cost of increasing production to 26 chairs monthly?

The cost of producing 26 chairs monthly would be:

C(26) = .001(26)^3 + .07(26)^2 + 19(26) + 700 = $1,258.896 ≈ $1,258.90

Therefore the additional cost to produce the 26th chair would be:

C(26) – C(25) = $1,258.90 – $1,234.38 = $24.52

c.   What is the marginal cost when x = 25?

C'(x) = .003x^2 + .14x + 19

C'(25) = .003(25)^2 + .14(25) + 19 = $24.375 ≈ $24.38

d.   Use marginal cost to estimate the difference in cost between producing 25 and 27 chairs per month.

Once 25 chairs have been produced, the cost to produce 2 more will be:

2($24.38) = $48.76

e.   Use the answer from part (d) to predict C(27).

C(27) = cost of producing 25 chairs + cost of producing 2 more chairs

C(27) = $1,234.38 + $48.76 = $1,283.14