3.6 An Economics Application: Elasticity of Demand

I. Introduction

Generally, if the price of an item rises, demand for it will usually fall. Retailers and manufacturers often need to know how a small change in price will affect the demand for a product. If a small increase in price produces no change in demand, a price increase make sense. But if a small increase in price creates a large drop in demand, a price increase is probably ill advised. To measure the sensitivity of demand to a small percent increase in price, economists calculate the **elasticity of demand.**

II. Definition of Elasticity of Demand

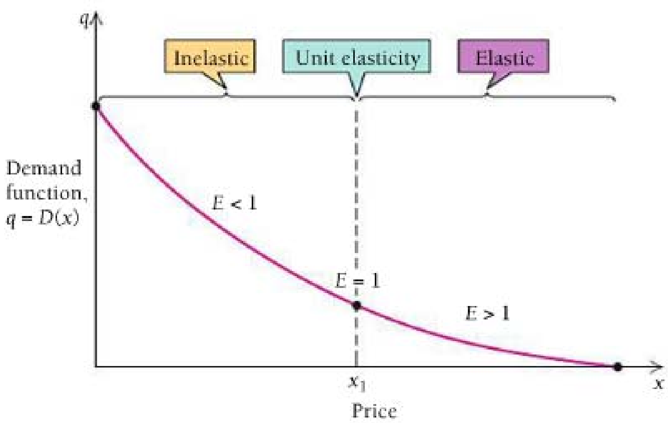
The **elasticity of demand** *E* is given as a function of price *x* by $E(x)=\dfrac{-x\cdot D'(x)}{D(x)}$.

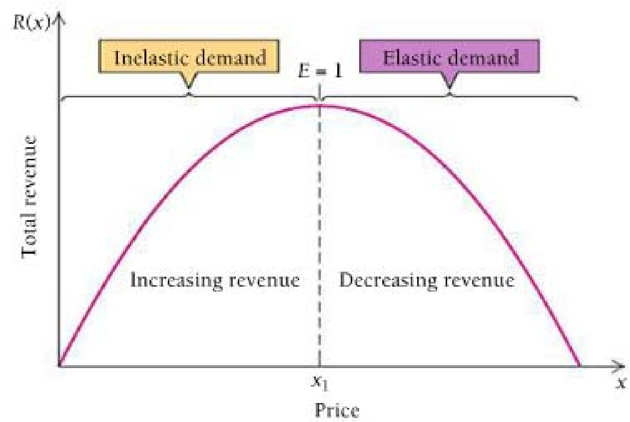
Intuitively, elasticity of demand is a measure of how responsive demand is to price changes.

III. Elasticity and Revenue (Theorem 15)

For a particular value of the price x:

1. The demand is **inelastic** if E(x) < 1. An increase in price will bring an increase in revenue. If demand is inelastic, then total revenue is increasing.
2. The demand has **unit elasticity** if E(x) = 1. The demand has unit elasticity when total revenue is at a maximum.
3. The demand is **elastic** if E(x) > 1. An increase in price will bring a decrease in revenue. If demand is elastic, then total revenue is decreasing.





Example 1: Given $q=D(x)=\sqrt{300-x}$ and x = 250, find the following:

a. the elasticity

b. the elasticity at the given price, stating whether the demand is elastic or inelastic

c. the value(s) of x for which total revenue is a maximum (assume that x is in dollars)

a. $D(x)=(300-x)^{\frac{1}{2}} \quad\rightarrow\quad D'(x)=\frac{1}{2}(300-x)^{-\frac{1}{2}}\cdot -1=-\frac{1}{2}(300-x)^{-\frac{1}{2}}$

$$E(x)=\frac{-x\cdot D'(x)}{D(x)} \quad\rightarrow\quad E(x)=\frac{-x\cdot -\frac{1}{2}(300-x)^{-\frac{1}{2}}}{(300-x)^{\frac{1}{2}}}=\frac{\frac{1}{2}x}{(300-x)^{\frac{1}{2}}\cdot(300-x)^{\frac{1}{2}}}=\frac{\frac{1}{2}x}{(300-x)}$$

$$=\frac{2}{2}\cdot\frac{\frac{1}{2}x}{(300-x)}=\frac{x}{600-2x}$$

b. $E(250)=\frac{250}{600-2(250)}=\frac{250}{600-500}=\frac{250}{100}=2.5$ Since E > 1, demand is elastic.

c. $\frac{x}{600-2x}=1$ → x = 600 – 2x → 3x = 600 → x = 200

A price of $200 will maximize total revenue.

Example 2: Demand for chocolate cookies

Good Times Bakers works out a demand function for its chocolate cookies and finds it to be q = D(x) = 967 – 25x where q is the quantity of cookies sold when the price per cookie, in cents, is x.

a. Find the elasticity.

b. At what price is elasticity of demand equal to 1?

c. At what price is elasticity of demand elastic?

d. At what price is elasticity of demand inelastic?

e. At what price is the revenue a maximum?

f. At a price of 20¢ per cookie, will a small increase in price cause the total revenue to increase or decrease?

a. $D'(x)=-25$ → $E(x)=\frac{-x\cdot -25}{967-25x}=\frac{25x}{967-25x}$

b. $\frac{25x}{967-25x}=1$ → 25x = 967 – 25x → 50x = 967 → $x=\frac{967}{50}=19.34\approx 19$

Demand is unitary elastic when the price per cookie is 19.34¢ ≈ 19¢.

c. Demand is elastic for prices greater than 19.34¢ ≈ 19¢.

d. Demand is inelastic for prices less than 19.34¢ ≈ 19¢.

e. Revenue is maximized when E(x) = 1 which occurs when the price is 19.34¢ ≈ 19¢.

f. At a price of 20¢ per cookie, demand is elastic, therefore a small increase in price will cause total revenue to decrease.