How to Graph  \( y = a \csc (bx - c) + d \) and  \( y = a \sec (bx - c) + d \)

1. Write the guide function. For  \( y = a \csc (bx - c) + d \) the guide function is  \( y = a \sin (bx - c) + d \) and for  \( y = a \sec (bx - c) + d \) the guide function is  \( y = a \cos (bx - c) + d \). 

2. Identify a, b, c, and d. 

3a. Find the amplitude of the guide function:  \( \text{amp} = |a| \).  \textbf{Note:} Cosecant and secant do not have amplitude. 

b. Determine if there has been an x-axis reflection: if a is negative, there has been an x-axis reflection. 

4a. Find the period:  \( \text{period} = \frac{2\pi}{b} \). 

b. Find the x-increment:  \( \text{x-increment} = \frac{1}{4} \cdot \text{period} \). 

5. Find the phase shift by setting the argument, \( bx - c \), equal to zero and solving for \( x \): 
\[ \text{P.S.} = \begin{cases} \frac{c}{b} \text{ right} & \text{if we see } bx - c \text{ and } \frac{c}{b} \text{ is positive} \\ \frac{c}{b} \text{ left} & \text{if we see } bx + c \text{ and } \frac{c}{b} \text{ is negative} \end{cases} \] 

6. Find the vertical shift:  \( |d| \) up if \( d \) is positive or  \( |d| \) down if \( d \) is negative. 

7. Identify the five key points of the guide function:
   for sine:  \( \left( \frac{c}{b}, d \right), \left( x_1 + \text{x-inc}, d + a \right), \left( x_2 + \text{x-inc}, d \right), \left( x_3 + \text{x-inc}, d - a \right), \left( x_4 + \text{x-inc}, d \right) \) 
   for cosine:  \( \left( \frac{c}{b}, d + a \right), \left( x_1 + \text{x-inc}, d \right), \left( x_2 + \text{x-inc}, d - a \right), \left( x_3 + \text{x-inc}, d \right), \left( x_4 + \text{x-inc}, d + a \right) \) 

8. Plot the five key points of the guide function and connect them with a smooth, continuous, dotted curve, then extend the graph to two full periods.

9. Draw the given function using these guidelines:
   Anywhere the guide function crosses its horizontal axis (i.e., when \( y = d \)), the given function has a vertical asymptote. 
   Anywhere the guide function has a minimum, the given function has a maximum and we draw a parabola shaped curve opening down. Anywhere the guide function has a maximum, the given function has a minimum, and we draw a parabola shaped curve opening up.

\textbf{Example 1: }  \( y = 2 \csc (x - 3\pi) - 1 \)

1. Guide function:  \( y = 2 \sin (x - 3\pi) - 1 \)

2.  \( a = 2 \)  \( b = 1 \)  \( c = 3\pi = \frac{6\pi}{2} \)  \( d = -1 \)

3.  \( \text{amp of sine} = |2| = 2 \)  There is no x-axis reflection.

4.  \( \text{period} = \frac{2\pi}{1} = 2\pi \)  \( \text{x-increment} = \frac{1}{4} \cdot 2\pi = \frac{2\pi}{4} = \frac{\pi}{2} \)

5.  \( \text{phase shift} = 3\pi \text{ or } \frac{6\pi}{2} \text{ right} \quad x - 3\pi = 0 \rightarrow x = 3\pi = \frac{6\pi}{2} \)

6.  \( \text{vertical shift} = 1 \text{ down} \)

7.  5 key points of the sine:  \( \left( \frac{6\pi}{2}, -1 \right), \left( \frac{7\pi}{2}, 1 \right), \left( \frac{8\pi}{2}, -1 \right), \left( \frac{9\pi}{2}, -3 \right), \left( \frac{10\pi}{2}, -1 \right) \)
Example 2: \( y = -\frac{1}{4} \sec \left(x + \frac{\pi}{2}\right) + 3 \)

1. Guide function: \( y = -\frac{1}{4} \cos \left(x + \frac{\pi}{2}\right) + 3 \)

2. \( a = -\frac{1}{4} \quad b = 1 \quad c = -\frac{\pi}{2} \quad d = 3 = \frac{12}{4} \)

3. amp of cosine = \( \left| \frac{-1}{4} \right| = \frac{1}{4} \) There is an x-axis reflection.

4. period = \( \frac{2\pi}{1} = 2\pi \) \( x \)-increment = \( \frac{1}{4} \cdot 2\pi = \frac{2\pi}{4} = \frac{\pi}{2} \)

5. phase shift = \( \frac{\pi}{2} \) left \( x + \frac{\pi}{2} = 0 \rightarrow x = -\frac{\pi}{2} \)

6. vertical shift = 3 or \( \frac{12}{4} \) up

7. 5 key points of the cosine:
   \[ \left( -\frac{\pi}{2}, \frac{11}{4} \right), \left( 0, \frac{12}{4} \right), \left( \frac{\pi}{2}, \frac{13}{4} \right), \left( \frac{2\pi}{2}, \frac{12}{4} \right), \left( \frac{3\pi}{2}, \frac{11}{4} \right) \]