Intermediate Algebra Projects –
MODELING IN THE REAL WORLD via data collection and analysis and algebraic methods.

PROBLEMS: (You won’t do all these at once, and in fact, may not do them all this semester. This list is designed to allow for variety of assignment but standardization of this form. Please note which ones are assigned when they are assigned.)

1) **Compound interest and credit card debt.**
   a) Find out the percent interest that one team member’s credit cards charges on an annual basis, the charge per month, and the minimum payment (or how to calculate it if it’s not a set amount) (if none of you use a credit card, ask a bank what they charge for theirs).
   b) (This may take several calculations. If you see a pattern or know of a formula, include that in your work and paragraph.) Calculate how much interest you will have paid while paying off an item if you charge $500 and pay
      i) only the minimum payment each month,
      ii) $10 over the minimum payment each month,
      iii) $50 over the minimum payment each month,
      iv) half the original charge amount each month.
   c) Discuss the results you found.

2) **Distance problems– floating down a river** – (only if time and weather allow).
   a) Your team needs to take a float trip down a local river. Make a measured string that is 100 meters long before going (also mark it for 100 feet). Use this to measure 100 meters at a point on the river where the flow is slow enough for your team to paddle up-stream (and the bank is clear enough to measure 100 meters in a straight line). (If you can’t find a place long-enough for 100 meters, use the 100-ft length.) Have two different pairs paddle up-stream separately. Time how long it takes each canoe to go up-stream the full 100 meters and then how long it takes them each to go down that same 100 meters.
   b) Separately, drop a floatable ball or other clearly visible easily catch-able object in the water at the head of the string and time how long it takes for it to float down to the foot of the string. Be sure to recapture the object at the foot! You may use a stick, as that would be less trouble (you can let it go on), but the data will be flawed if it gets hung up somewhere.
   c) Once home again, use the formula relating distance, rate, and time: distance = rate \times time, and the data from part (a) only to calculate the rate for each team in still water and the rate of the river’s current. If the current is not the same for all canoes, try to state why in your paragraph.
   d) Use the formula relating distance, rate, and time: distance = rate \times time, and the data from part (b) only to calculate the river’s current.
   e) Compare and contrast the current you found for the river in parts (c) and (d). If they’re different, try to decide why that might happen.
3) **Work problems** (examples can be found in Ch 6).

Formulas: \((\text{rate-on-task}) \times (\text{time-on-task}) = 1\) task completed, so \(\text{rate} = \frac{1}{\text{time on task}}\).

These types of rates will add up (working together gets the job done faster than each working alone), so: \(\text{rate}_1 + \text{rate}_2 = (\text{rates together})\). Combining these two concepts gives us a standard work formula for calculating time when each person, object, or machine is doing the same job

\[ \Rightarrow \frac{1}{\text{time}_1} + \frac{1}{\text{time}_2} = \frac{1}{\text{time together}}. \]

Commonly, the formula will take on this simpler form: \(\frac{1}{a} + \frac{1}{b} = \frac{1}{t}\), where \(a\) and \(b\) are time for each person or machine to do the task separately and \(t\) is the time together (in both formulas). With this in mind, complete the following three problems.

a) **Data collection and comparison with the formula:**
   i) Determine a task at least two of your teammates can accomplish alone and/or together (mow a lawn with two mowers, paint a wall with enough equipment for two people, etc). ASK me before starting, if you are not sure your chosen task will fit this process calculation or not (e.g., one example which won’t work is to read a page in a book, since it doesn’t take less time for each to read it separately vs. together).
   ii) Time the two team members performing the entire task separately.
   iii) Time them working on the task together.
   iv) Use the work formula (above) to calculate what their time should be together based on their separate times. Compare (and contrast) the two results your team gets from the actual timing together vs. what the formula predicts their time should be together in your paragraph.

b) **Solve:** It takes two people 3 hr and 4 hr separately to do a task. Find how long it would take them to do the same task together.

c) **Solve:** It takes two copiers 10 minutes and 25 minutes to copy a 500 page project separately. Find how long it would take using both copiers to get one 500 page project copied.

4) **Using similar triangles to estimate the height of tall objects.** Fact: similar triangles have the same angle measures and their sides are proportional:

\[
\begin{align*}
30 \text{ mm} &\quad \begin{array}{c}30 \text{ mm} \end{array} \\
34 \text{ mm} &\quad \begin{array}{c}34 \text{ mm} \end{array}
\end{align*}
\]

\[
\frac{30}{23} = \frac{34}{26} \quad \text{and} \quad \frac{26}{34} = \frac{23}{30}
\]

are just two of several appropriate ratios we can form.

a) In the following, it’s easier if you use one type of unit of measure (either all in feet or all in inches) for the measurements (but the building’s measurement unit can be different than the person’s). You will need to bring a long tape measure the day you do this (I recommend at least 30 ft, but if necessary, you may borrow my 25 ft long one). Also, recall 1 ft = 12 in, so to convert to feet, divide a number of inches by 12.

b) On a sunny day, measure one team-member’s height and the length of their shadow on the ground carefully.

c) At that same time, measure the shadow of WLF or MAT (state which you are using) where a shadow’s corner or peak is SHARP and easy to see.

\[
\begin{array}{c}
\text{Person} \quad \text{Building}
\end{array}
\]

\[
\begin{array}{c}
\text{Shadow} \quad \text{Shadow}
\end{array}
\]

d) Use these three measurements and similar triangles (see the diagram above) to estimate the height of the building at that corner.

e) Compare your results with another team in the class. Comment on any similarities or differences in your paragraph.
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5) Pythagorean Theorem. Given a right triangle, with legs $a$ and $b$ and hypotenuse $c$, then

\[ a^2 + b^2 = c^2 \]

i.e., \((\text{leg}_1)^2 + (\text{leg}_2)^2 = (\text{hypotenuse})^2\)

\[
\begin{align*}
\text{leg}_1 &= a \\
\text{leg}_2 &= b \\
\text{hypotenuse} &= c
\end{align*}
\]

a) Using the Pythagorean Theorem in Measurement: Someone from your team will need to bring in or borrow a 25ft or 30ft measuring tape. Use this to measure two straight sides of our class-room accurately. Use the Pythagorean theorem to find the diagonal measurement. Find this measurement both exactly and to the nearest three decimal places. Also measure the diagonal using the measuring tape (if possible). Decide if the room has “squared corners.”

b) Pythagorean Triplets: As a separate problem, find the first five Pythagorean Triplets. (A Pythagorean Triplet is any three whole numbers $a$, $b$, and $c$ such that $c^2 = a^2 + b^2$. ) (Hint, you might try an internet search.)

c) Connections: Pythagorean Theorem and the formula for the Distance Between any Two Points: As a separate problem, use the formula for the Distance Between any Two Points and general right triangle with vertical and horizontal legs in a Cartesian coordinate plane. Name the vertices on the ends of the hypotenuse $(x_1, y_1)$ and $(x_2, y_2)$ and vertex at the right angle $(x_2, y_1)$. Name the horizontal leg $a$ and vertical leg $b$ and hypotenuse $c$. Now, find the lengths of $a$ and $b$ in terms of $(x_1, y_1)$, $(x_2, y_1)$, and $(x_2, y_2)$. Then use the Pythagorean Theorem and the lengths of each leg via the coordinates to find the distance along the hypotenuse.

6) Function using Revenue and Cost. Given the following revenue and cost data, find each of the following. In each case, $x =$ number of units produced (cost) or sold (revenue).

Profit Functions: \[ P(x) = [ R(x) ] - [ C(x) ] \]

Break-even point: This occurs when Revenue = Cost

Set $R(x) = C(x)$ to find the number of units produced and sold, then find $R(x)$ to find the revenue amount (which is the same as the cost amount here, hence the name break-even). Use these both to state the ordered pair of the break-even point.

a) Suppose you have a business that makes widgets. Your fixed costs are $5000 (rent, utilities, and so on) and it costs $10 per widget to produce each widget (for materials and equipment depreciation and so on). You decide to sell each widget for $20.

i) Find your total cost function, $C(x)$.

ii) Find your total revenue function, $R(x)$.

iii) Find your total profit function, $P(x)$.

iv) Find your break-even point. Remember to explain what this means.

b) Suppose you want to start a small business near one team-member’s home (your team decides what business, but it needs to produce an item or service for sale.)

i) Find out these fixed costs for a storefront near one team-member’s home: rent, utilities.

ii) Estimate labor at $10/hr for just two employees to start (yourself and one other), and start-up equipment costs at $10,000.

iii) Estimate materials costs and labor per item produced at $10,000

iv) Set the price of the product/service at $5 per unit.

v) Find your total cost function, $C(x)$.

vi) Find your total revenue function, $R(x)$.

vii) Find your total profit function, $P(x)$.

viii) Find your break-even point. Remember to explain what this means.
PROBLEMS: (You won’t do all these at once, and in fact, may not do them all this semester. This list is designed to allow for variety of assignment but standardization of this form. Please note which ones are assigned when they are assigned.)

7) An object thrown/shot into the air: The function \( s(t) = -16t^2 + v_0t + s_0 \) measures the height of an projectile where \( s(t) = s = \text{height (in feet) at time } t \text{ (in seconds)}, \) \( v_0 \) is the initial velocity (speed the object is thrown/shot) (in feet per second or ft/sec) and \( s_0 \) is the initial height (in feet).

General Description: Using a tennis ball, one team member will throw it up into the air as hard as possible a few times, while other team members find the initial height and track the time the ball is airborne. The team will use these to find the initial velocity for each throw and then the average throwing velocity. The graph of the team’s general formula will also be seen on a graphing calculator, and then sketched on graph paper.

- **Release (initial) Height**
  - \( s_0 \) (in feet)
- **Height when projectile hits the ground**
  - \( s \) (in feet)
- **Airborne Time**
  - \( t \) (in seconds)
- **Height when projectile hits the ground**
  - \( s \) (in feet)
- **Throwing velocity for each throwing time**
  - \( v \) (in ft/sec)
- **Average throwing velocity**
  - \( v_0 \) (Use this as \( v_0 \) in all remaining calculations)
- **Table of coordinate pairs comparing (Time, Height) for the function using the average velocity found.**
  - \((t, s(t))\)
  - \((0, 0)\)
  - \(\left(\frac{1}{2}, \frac{1}{2}\right)\) (Vertex)
  - \(\left(2, 2\right)\)

a) Using one practice throw, have the thrower keep their hand up as high as it was when they released the ball, and the other team members quickly and carefully measure how high that is off the ground. Only measure this and record it as the initial height measurement once in a Data Table (see example above). Be sure to record feet and inches carefully, and then convert the inches part to feet form (as an exact, reduced, fraction or an exact decimal), before working on other calculations using the formula.

b) Now, using a stop watch (borrow one from me if you don’t have one in the team), time 5 separate throws accurately. For each throw, start the watch when the thrower releases and stop it when the ball hits the ground. Record that airborne time to the nearest hundredth of a second in your Data Table. Repeat this until 5 throwing-to-ground times are recorded. Each of these times is a separate \( t \)-value.

c) Now for a little bit of algebra: given the height formula, when the ball hits the ground, what is the height (from the ground)? Replace \( s(t) \) (on the left side) by that value in the formula. \( s(t) = -16t^2 + v_0t + s_0 \)

It will be easier to work with if this new formula is solved for throwing velocity \( (v_0) \). Solve it for \( v_0 \). Be sure to record a general copy of that formula.

d) Now, use your first \( t \) and your \( s_0 \) and place them into the new formula you found in part (c). Write that complete expression in the Data Table. Calculate the \( v_0 \) and write it in the Data Table. Repeat this process for the second \( t \), and so on until you’ve used all five \( t \)-s. Remember to use correct order-of-operations by keying in the entire expression (parentheses in place and all) carefully each time.

e) Now find the average initial velocity for your team’s thrower, and put it in your Data Table.

f) Next, write your team’s general height function by replacing both \( s_0 \) and \( v_0 \) by your one \( s_0 \) and the average \( v_0 \):

\[ s(t) = -16t^2 + v_0t + s_0 \rightarrow s(t) = -16t^2 + \text{average } t + \text{constant}. \]

g) Use your general height function to find each of the following points. Put all these points in a handwritten t-table (with your Data Table).

i) The time and height at the initial height (hint, when you throw, how much time has elapsed?).

ii) The time and height when the ball hits the ground (hint, when the ball hits the ground, how high is it?)

iii) The time and height at the highest point (the vertex).

iv) The time when the ball was 15 ft high (hint, let \( s(t) = 15 \) and solve the equation this creates, this should give you two solutions which gives you two points).

h) Use all these points to create a SCATTERPLOT of your data.
To see this graph on a TI-82/83 graphing calculator, press ON Y= (–) 1 6 X^0 + v_0 X^0 + s_0.

(Be sure to key in your average \(v_0\) and \(s_0\) numbers where those are written here.)

Window ENTER 0 ENTER 6 ENTER ENTER 0 ENTER 6 ENTER 5 ENTER 1 0 ENTER GRAPH
(Xmin = 0; Xmax = 6; Xscl = 1, Ymin = 0; Ymax = 65; Yscl = 10)

j) Sketch the continuous (full) graph of the function you wrote in part (f) on the same Cartesian coordinate plane as your scatterplot. (Use a different color than the scatterplot so anyone can tell them apart.) Compare and contrast the scatter plot to the graph of the function \(s(t)\). Try to describe what each point in the scatterplot represents.

8) Web-research

a) Calculating escape velocity

Escape velocity is the minimum speed that an object must reach to escape a planet’s pull of gravity. Escape velocity \(v\) is given by the equation \(v = \sqrt{\frac{2Gm}{r}}\), where \(m\) is the mass of the planet, \(r\) is the radius, and \(G\) is the universal gravitational constant, which has a value of \(G = 6.67 \times 10^{-11}\) m\(^3\)/(kg \cdot s\(^2\)).

Go to this website: [http://nssdc.gsfc.nasa.gov/planetary/planetfact.html](http://nssdc.gsfc.nasa.gov/planetary/planetfact.html)

Choose any one of the fact sheets for a planet, asteroid, or the sun. Use the information given in the fact sheet and the escape velocity formula given above to compute the escape velocity for that body. Then compare your calculation to the escape velocity given in the fact sheet. How close is your calculation? (Note: use the “volumetric mean radius” for the planet’s radius in your calculation.) State the answer (in words).

b) Compound Interest (annual compounding)

The formula for calculating the total amount, \(A\), of money when interest is compounded annually is \(A = P(1 + r)^t\) where \(P\) is the original investment, \(r\) is the interest rate per compounding period, and \(t\) is the number of periods (here in years). For example, the amount of money \(A\) at the end of 3 years if $100 is invested at 3% compounded annually is \(A = 100(1 + 0.03)^3 = 100(1.092727) = 109.27\).

i) Use this formula to find the rate \(r\) at which $3000 grows to $4320 in 2 years.


This World Wide Web address will direct you to a Web site that contains current interest rates - both composite rates and those offered by individual savings institutions – on a wide variety of financial products such as savings deposits and auto loans.

Choose a financial product. Using actual data for a current interest rate on that type of product, write a problem similar to exercise a) above. Solve the problem. Check your work. State the answer (in words).